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Phenomenology Working Group

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Abstract

The Phenomenology Working Group at NOW'98 discussed the following topics:

- Possible interpretations of neutrino oscillation data in the framework of neutrino mixing.
- Phenomenological models that accommodate the neutrino oscillation data.
- Ideas for future experiments.
- The role of neutrinos in cosmology.
- Quantum-mechanical problems of neutrino oscillations.
- Problems of the statistical interpretation of the data.

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1 Neutrino Mixing and Oscillations — Samoil Bilenky

1.1 Introduction

The neutrino mixing hypothesis [1–5] (see also [6–9]) is based on the assumption that neutrino masses are different from zero and the neutrino mass term does not conserve lepton numbers. In this case, the left-handed flavor neutrino fields $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau$) are superpositions of the left-handed components ν_{kL} ($k = 1, \dots, n$) of the fields of neutrinos with definite masses m_k :

$$\nu_{\alpha L} = \sum_{k=1}^n U_{\alpha k} \nu_{kL}, \quad (1.1)$$

where U is a unitary $n \times n$ mixing matrix. The left-handed flavor neutrino fields $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau$) are determined by charged-current (CC) and neutral-current (NC) weak interactions with Lagrangians

$$\mathcal{L}_I^{\text{CC}} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \alpha_L W^\rho + \text{h.c.}, \quad (1.2)$$

$$\mathcal{L}_I^{\text{NC}} = -\frac{g}{2 \cos \theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \nu_{\alpha L} Z^\rho + \text{h.c.} \quad (1.3)$$

From the precise measurement of the invisible width of the decay of the Z -boson we know that the number of light flavor neutrinos is equal to three (see [10]), corresponding to ν_e , ν_μ and ν_τ . On the other hand, the number n of massive neutrinos has no experimental constraint, besides being bigger or equal than three.

There are two possibilities for the fundamental nature of massive neutrinos:

Dirac. If the total lepton number

$$L \equiv L_e + L_\mu + L_\tau \quad (1.4)$$

is conserved because of the invariance of the Lagrangian under the global gauge transformation

$$\nu_\alpha \rightarrow e^{i\varphi} \nu_\alpha, \quad \alpha \rightarrow e^{i\varphi} \alpha \quad (\alpha = e, \mu, \tau), \quad (1.5)$$

then massive neutrinos are Dirac particles. In this case:

- The fields ν_k have four independent complex components.
- It is natural to expect that the number n of massive neutrinos is equal to the number of flavor neutrinos, *i.e.* three, although nothing forbids in principle the existence of sterile Dirac neutrinos.
- Neutrinoless double- β decay $((\beta\beta)_{0\nu})$ is forbidden.
- Dirac masses and mixing can be generated with the Higgs mechanism of the Standard Model.

Majorana. If the Lagrangian is not invariant under the global gauge transformation (1.5), the total lepton number L is not conserved and massive neutrinos are Majorana particles, *i.e.* truly neutral fermions which do not have any charge (electric, leptonic, etc.) that distinguishes particle from antiparticle. In this case:

- The massive Majorana fields ν_k satisfy the Majorana condition

$$\nu_k = \nu_k^c, \quad (1.6)$$

where $\nu_k^c \equiv \mathcal{C}\overline{\nu_k}^T$ and \mathcal{C} is the charge-conjugation matrix.

- Neutrinoless double- β decay is allowed.
- If right-handed neutrino fields ν_{aR} (singlets of $SU(2)_L$) exist, the number n of massive Majorana neutrino is bigger than three. In this case to the mixing relations (1.1) one must add the relations between the right-handed fields ν_{aR} and the massive fields ν_k :

$$\nu_{aR}^c = \sum_{k=1}^n U_{ak} \nu_{kL}. \quad (1.7)$$

The quanta of the right-handed fields are sterile neutrinos that do not participate to weak interactions.

In the Majorana case there are two plausible options:

- (I) The *see-saw* option [11]. If the total lepton number is violated by the right-handed Majorana mass term at an energy scale much larger than the electroweak scale, the Majorana mass spectrum is composed by three light masses m_k ($k = 1, 2, 3$) and three very heavy masses M_k ($k = 1, 2, 3$) that characterize the scale of lepton number violation. In the simplest see-saw scenario (see, for example, [8, 12–15] and references therein) the light neutrino masses are given by

$$m_k \sim \frac{(m_k^F)^2}{M_k} \ll m_k^F \quad (i = 1, 2, 3). \quad (1.8)$$

where m_k^F is the mass of the charged lepton or up-quark in the k^{th} generation. The see-saw mechanism provides a plausible explanation for the smallness of neutrino masses with respect to the masses of all other fundamental fermions.

- (II) The *sterile neutrino* option. If more than three Majorana mass terms are small, then there are light sterile neutrinos. In this case active neutrinos ν_e , ν_μ and ν_τ can oscillate into sterile states. Notice that sterile neutrinos can be obtained in the framework of the see-saw mechanism with some additional assumptions (“singular see-saw” [16], “universal see-saw” [17]).

The main open problems concerning neutrinos are:

1. What are the values of

- (a) the neutrino masses m_k ?
- (b) the elements $U_{\alpha k}$ of the mixing matrix?
- 2. Which is the nature of massive neutrinos (Dirac or Majorana)?
- 3. Which is the number of massive neutrinos?
- 4. Can active neutrinos oscillate into sterile states?
- 5. Is CP violated in the lepton sector?

1.2 Neutrino Oscillations

A neutrino of flavor α and momentum \vec{p} produced in a weak interaction process is described by the state

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle. \quad (1.9)$$

Here $|\nu_k\rangle$ is a state describing a massive neutrino with momentum \vec{p} and energy

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p}, \quad (1.10)$$

where $p \equiv |\vec{p}|$. The expression (1.9) is based on the assumption that *the state of a flavor neutrino is a coherent superposition of states of neutrinos with different masses*.

The general expression for the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions in vacuum can be written as

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \delta_{\alpha\beta} + \sum_{k=2}^n U_{\alpha k}^* U_{\beta k} \left[\exp\left(-i \frac{\Delta m_{k1}^2 L}{2E}\right) - 1 \right] \right|^2, \quad (1.11)$$

where $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$, L is the distance between the neutrino source and detector and $E \simeq p$ is the neutrino energy. It is clear that neutrino oscillations can be observed only if there is at least one Δm_{kj}^2

$$\frac{\Delta m_{kj}^2 L}{E} \gtrsim 1. \quad (1.12)$$

There are three experimental indications in favor of neutrino oscillations coming from the results of

1. Atmospheric neutrino experiments (Super-Kamiokande [18], Kamiokande [19], IMB [20], Soudan [21]) with the squared mass difference

$$\Delta m_{\text{atm}}^2 \sim 3 \times 10^{-3} \text{ eV}^2. \quad (1.13)$$

2. Solar neutrino experiments (Homestake [22], Kamiokande [23], GALLEX [24], SAGE [25], Super-Kamiokande [26]) with

$$\Delta m_{\text{sun}}^2 \sim 10^{-5} \text{ eV}^2 \quad (\text{MSW [27]}), \quad (1.14)$$

or

$$\Delta m_{\text{sun}}^2 \sim 10^{-10} \text{ eV}^2 \quad (\text{vacuum osc. [5]}). \quad (1.15)$$

3. The LSND experiment [28], with

$$\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2. \quad (1.16)$$

Furthermore, in order to extract from the experimental data information on the values of the neutrino masses and mixing angles it is necessary to take into account also the negative results of numerous reactor and accelerator short-baseline experiments (the latest and most restrictive ones are: Bugey [29] for the $\nu_e \rightarrow \nu_e$ channel, CDHS [30] and CCFR [31] for the $\nu_\mu \rightarrow \nu_\mu$ channel, BNL E776 [32], CCFR [33] and KARMEN [34] for the $\nu_\mu \rightarrow \nu_e$ channel, CHORUS [35] and NOMAD [36] for the $\nu_\mu \rightarrow \nu_\tau$ channel) and of the recent CHOOZ reactor long-baseline experiment [37].

From Eqs.(1.13)–(1.16) the experimental results indicate the existence of three different scales of Δm^2 , *i.e.* at least four massive neutrinos [38–42] (see also Section 2. The four types of neutrino mass spectra that can accommodate the solar, atmospheric and LSND scales of Δm^2 are shown in Fig. 1.1. In all these spectra there are two groups of close masses separated by a gap of the order of 1 eV which provides the mass-squared difference $\Delta m_{\text{LSND}}^2 = \Delta m_{41}^2 \equiv m_4^2 - m_1^2$ that is relevant for the oscillations observed in the LSND experiment. Two years ago we have shown [39] that, if also the negative results of numerous short-baseline neutrino oscillation experiments are taken into account, among the possible schemes shown in Fig. 1.1 only the schemes A and B are compatible with the results of all neutrino oscillation experiments. In scheme A $\Delta m_{\text{atm}}^2 = \Delta m_{21}^2 \equiv m_2^2 - m_1^2$ is relevant for the explanation of the atmospheric neutrino anomaly and $\Delta m_{\text{sun}}^2 = \Delta m_{43}^2 \equiv m_4^2 - m_3^2$ is relevant for the suppression of solar ν_e 's, whereas in scheme B $\Delta m_{\text{atm}}^2 = \Delta m_{43}^2$ and $\Delta m_{\text{sun}}^2 = \Delta m_{21}^2$. These two schemes have important consequences for long-baseline experiments [40], for the possibility to observe CP violation in the lepton sector [41] and for Big-Bang nucleosynthesis [42, 43]. The phenomenology of neutrino oscillations in the schemes A and B is identical, but in the scheme A the effective neutrino masses in Tritium β -decay experiments and in neutrinoless double- β decay experiments can be of the order of $m_3 \simeq m_4$, whereas in the scheme B they are strongly suppressed [39]. Hence, the observation of the effect of neutrino masses of the order of 0.1 – 1 eV in Tritium β -decay experiments and in neutrinoless double- β decay experiments could allow to distinguish between the two schemes, favoring the scheme A.

1.3 Mixing of three neutrinos

If the results of the LSND experiment will not be confirmed by future experiments, the most plausible scheme is the one with mixing of three massive neutrinos and a mass hierarchy:

$$m_1 \ll m_2 \ll m_3. \quad (1.17)$$

In this case $\Delta m_{\text{sun}}^2 = \Delta m_{21}^2 \equiv m_2^2 - m_1^2$ is relevant for the suppression of the flux of solar ν_e 's and $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2 \equiv m_3^2 - m_1^2$ is relevant for the atmospheric neutrino anomaly.

The probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions is given by

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_{k=1}^3 U_{\alpha k}^* U_{\beta k} \exp \left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2, \quad (1.18)$$

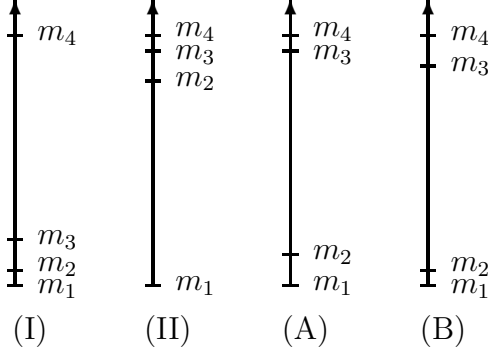


Figure 1.1

Figure 1.1: The four types of neutrino mass spectra that can accommodate the solar, atmospheric and LSND scales of Δm^2 .

where E is the neutrino energy and L is the distance between the neutrino source and detector. Let us consider vacuum oscillations in atmospheric and long-baseline (LBL) neutrino oscillation experiments. Taking into account that in these experiments

$$\frac{\Delta m_{21}^2 L}{2E} \ll 1 \quad (1.19)$$

and using the unitarity of the mixing matrix, we obtain

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{LBL}} = \left| \delta_{\alpha\beta} + U_{\beta 3} U_{\alpha 3}^* \left[\exp \left(-i \frac{\Delta m_{31}^2 L}{2E} \right) - 1 \right] \right|^2. \quad (1.20)$$

Thus, under the condition (1.19), the atmospheric and LBL transition probabilities in vacuum are determined only by the largest mass squared difference Δm_{31}^2 and by the elements of the mixing matrix that connect flavor neutrinos with the heaviest neutrino ν_3 .

From the expression (1.20), for the probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions with $\beta \neq \alpha$ and for the survival probability of ν_α we find

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{LBL}} = \frac{1}{2} A_{\beta;\alpha} \left(1 - \cos \frac{\Delta m_{31}^2 L}{2E} \right), \quad \text{for } \beta \neq \alpha, \quad (1.21)$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha}^{\text{LBL}} = 1 - \frac{1}{2} B_{\alpha;\alpha} \left(1 - \cos \frac{\Delta m_{31}^2 L}{2E} \right) \quad (1.22)$$

with the oscillation amplitudes $A_{\beta;\alpha}$ and $B_{\alpha;\alpha}$ given by

$$A_{\beta;\alpha} = 4 |U_{\beta 3}|^2 |U_{\alpha 3}|^2, \quad (1.23)$$

$$B_{\alpha;\alpha} = \sum_{\beta \neq \alpha} A_{\beta;\alpha} = 4 |U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2). \quad (1.24)$$

Hence, in the case of a hierarchy of the masses of three neutrinos, neutrino oscillations in atmospheric and LBL experiments are characterized by only one oscillation length.

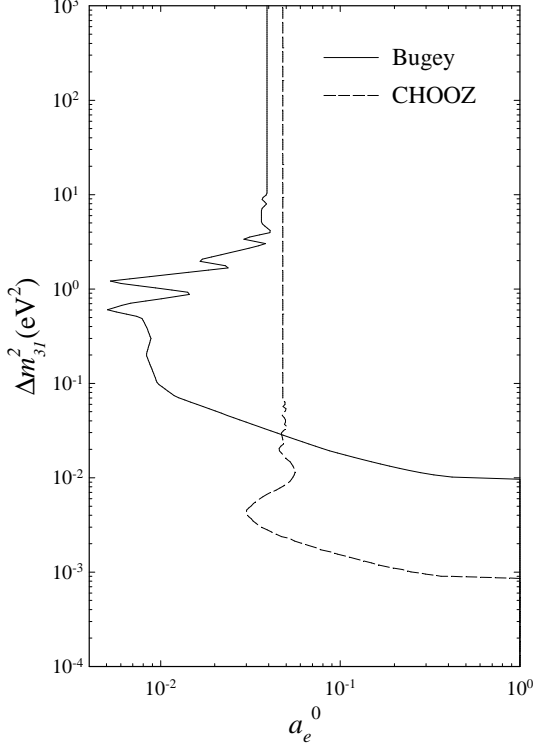


Figure 1.2

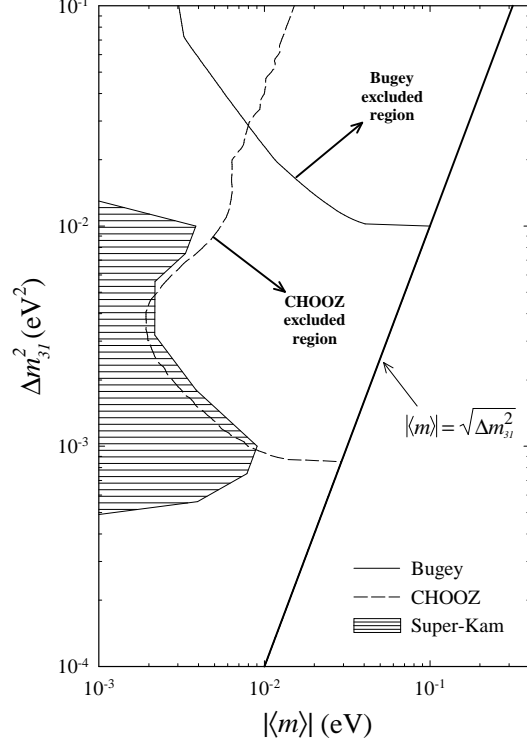


Figure 1.3

Furthermore, from Eqs.(1.21) and (1.22) one can see that the dependence of the transition probabilities on the quantity $\Delta m_{31}^2 L/2E$ has the same form as in the standard two-neutrino case. Let us stress, however, that the expressions (1.21) and (1.22) describe transitions between all three flavor neutrinos. Notice also that the transition probabilities in atmospheric and LBL experiments do not depend on the possible CP-violating phase in the mixing matrix and we have

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{LBL}} = P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}^{\text{LBL}} \quad (1.25)$$

As it is seen from Eqs.(1.21)–(1.24), in the scheme under consideration the oscillations in all channels ($\nu_e \leftrightarrow \nu_\mu$, $\nu_\mu \leftrightarrow \nu_\tau$, $\nu_e \leftrightarrow \nu_\tau$) are described by three parameters: Δm_{31}^2 , $|U_{e3}|^2$, $|U_{\mu 3}|^2$ (because of unitarity of the mixing matrix $|U_{\tau 3}|^2 = 1 - |U_{e3}|^2 - |U_{\mu 3}|^2$).

With the help of Eqs.(1.22) and (1.24), one can obtain bounds on the mixing parameter $|U_{e3}|^2$ from the exclusion plots obtained in the Bugey reactor experiment [29] and in the recent CHOOZ experiment [44], which is the first reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ LBL experiment. At any fixed value of Δm_{31}^2 in the range explored by the CHOOZ experiment we obtain the upper bound $B_{ee} \leq B_{ee}^0$ for $\alpha = e, \mu$. From Eq.(1.24), for the mixing parameter $|U_{e3}|^2$ we have

$$|U_{e3}|^2 \leq a_e^0 \quad \text{or} \quad |U_{e3}|^2 \geq 1 - a_e^0, \quad \text{with} \quad a_e^0 = \frac{1}{2} \left(1 - \sqrt{1 - B_{ee}^0} \right). \quad (1.26)$$

In Fig. 1.2, we have plotted the values of the parameter a_e^0 obtained from the 90% CL exclusion plots of the Bugey and CHOOZ experiments. One can see that a_e^0 is very small for $\Delta m_{31}^2 \gtrsim 10^{-3} \text{ eV}^2$. Thus, the results of the reactor oscillation experiments imply that $|U_{e3}|^2$ can only be small or large (close to one).

Now let us take into account the results of solar neutrino experiments. The probability of solar neutrinos to survive in the case of a neutrino mass hierarchy is given by [45]

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) = (1 - |U_{e3}|^2)^2 P_{\nu_e \rightarrow \nu_e}^{(1,2)}(E) + |U_{e3}|^4, \quad (1.27)$$

where E is the neutrino energy and $P_{\nu_e \rightarrow \nu_e}^{(1,2)}(E)$ is the two-generation survival probability of solar ν_e 's. If $|U_{e3}|^2 \geq 1 - a_e^0$, from (1.27) it follows that at all solar neutrino energies $P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \gtrsim 0.92$. This is not compatible with the results of solar neutrino experiments. Thus, the mixing parameter $|U_{e3}|^2$ must be small:

$$|U_{e3}|^2 \leq a_e^0. \quad (1.28)$$

1.3.1 Decoupling of solar and atmospheric neutrino oscillations

Let us consider the solar and atmospheric neutrino anomalies assuming that Δm_{21}^2 is responsible for the oscillations of solar neutrinos and Δm_{31}^2 is responsible for the oscillations of atmospheric neutrinos (here we follow the discussion presented in Ref. [46]).

From Fig. 1.2 one can see that, if $\Delta m_{31}^2 > 10^{-3} \text{ eV}^2$ as indicated by the solution of the Kamiokande atmospheric neutrino anomaly, [19], the results of the CHOOZ experiment implies that

$$|U_{e3}|^2 \leq 5 \times 10^{-2}. \quad (1.29)$$

The averaged survival probability of solar electron neutrinos is given by Eq.(1.27). Taking into account the small upper bound (1.29) for $|U_{e3}|^2$, we have

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \simeq P_{\nu_e \rightarrow \nu_e}^{(1,2)}(E), \quad (1.30)$$

where $P_{\nu_e \rightarrow \nu_e}^{(1,2)}(E)$ is the two-generation survival probability of solar ν_e 's which depends on

$$\Delta m_{\text{sun}}^2 = \Delta m_{21}^2 \quad \text{and} \quad \sin \vartheta_{\text{sun}} = \frac{|U_{e2}|}{\sqrt{1 - |U_{e3}|^2}} \simeq |U_{e2}|. \quad (1.31)$$

Hence the two-generation analyses of the solar neutrino data are appropriate in the three-neutrino scheme with a mass hierarchy and they give information on the values of

$$\Delta m_{21}^2 = \Delta m_{\text{sun}}^2 \quad \text{and} \quad |U_{e2}| \simeq \sin \vartheta_{\text{sun}}. \quad (1.32)$$

The evolution equation for the flavor amplitudes ψ_α ($\alpha = e, \mu, \tau$) of atmospheric neutrinos propagating in the interior of the Earth can be written as (see [47, 48])

$$i \frac{d}{dt} \Psi = \frac{1}{2E} (U M^2 U^\dagger + A) \Psi, \quad (1.33)$$

with

$$\Psi \equiv \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}, \quad M^2 \equiv \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2), \quad A \equiv \text{diag}(A_{CC}, 0, 0), \quad (1.34)$$

and $A_{CC} \equiv 2EV_{CC}$, where $V_{CC} = \sqrt{2}G_F N_e$ is the charged-current effective potential which depends on the electron number density N_e of the medium (G_F is the Fermi

constant and for anti-neutrinos A_{CC} must be replaced by $\bar{A}_{CC} = -A_{CC}$). If the squared-mass difference Δm_{21}^2 is relevant for the explanation of the solar neutrino problem, we have

$$\frac{\Delta m_{21}^2 R_{\oplus}}{2E} \ll 1, \quad (1.35)$$

where $R_{\oplus} = 6371$ Km is the radius of the Earth. Notice, however, that caution is needed for low-energy atmospheric neutrinos if $\Delta m_{21}^2 \gtrsim 10^{-5} \text{ eV}^2$, as in the case of the large mixing angle MSW solution of the solar neutrino problem and marginally in the case of the small mixing angle MSW solution. Indeed, if $\Delta m_{21}^2 \gtrsim 10^{-5} \text{ eV}^2$ we have $\Delta m_{21}^2 R_{\oplus}/2E \ll 1$ only for $E \gg 150$ MeV. In this case, in order to get information on the three-neutrino mixing matrix with a two-generation analysis it is necessary to analyze the atmospheric neutrino data with a cut in energy such that $\Delta m_{21}^2 R_{\oplus}/2E \ll 1$. In order to be on the safe side, when the case of the MSW solutions of the solar neutrino problem are considered one can take into account the information obtained from the two-generation fit of the SuperKamiokande multi-GeV data alone [46].

The inequalities (1.35) imply that the phase generated by Δm_{21}^2 can be neglected for atmospheric neutrinos and M^2 can be approximated with

$$M^2 \simeq \text{diag}(0, 0, \Delta m_{31}^2). \quad (1.36)$$

In this case (taking into account that the phases of the matrix elements $U_{\alpha 3}$ can be included in the charged lepton fields) we have

$$(U M^2 U^\dagger)_{\alpha' \alpha} \simeq \Delta m_{31}^2 |U_{\alpha' 3}| |U_{\alpha 3}|. \quad (1.37)$$

Comparing this expression with Eqs.(1.30) and (1.32), one can see that the oscillations of solar and atmospheric neutrinos depend on different and independent Δm^2 's and on different and independent elements of the mixing matrix, *i.e.* they are decoupled. Strictly speaking $|U_{e2}|$ in Eqs.(1.30) and (1.32) is not independent from $|U_{e3}|$ because of the unitarity constraint $|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$, but the limit (1.28) on $|U_{e3}|^2$ implies that its contribution to the unitarity constraint is negligible.

Hence, we have shown that the smallness of $|U_{e3}|^2$ inferred from the results of the CHOOZ experiment imply that *the oscillations of solar and atmospheric neutrinos are decoupled* [46].

From Eqs.(1.33) and (1.37) one can see that unless $|U_{e3}| \ll 1$, the evolution equations of the atmospheric electron neutrino amplitude ψ_e and those of the muon and tau neutrino amplitudes ψ_μ and ψ_τ are coupled. In this case matter effects can contribute to the dominant $\nu_\mu \rightarrow \nu_\tau$ oscillations (see [49]) and the atmospheric neutrino data must be analyzed with the three generation evolution equation (1.33).

From the results of the CHOOZ experiment it follows that the quantity $|U_{e3}|^2$ is small and satisfy the inequality (1.28) (for $\Delta m_{31}^2 \gtrsim 10^{-3} \text{ eV}^2$). However, the upper bound for $|U_{e3}|$ implied by Eq.(1.28) is not very strong: $|U_{e3}| < 0.22$. In the following we will assume that not only $|U_{e3}|^2 \ll 1$, but also the element $|U_{e3}|$ that connects the first and third generations is small: $|U_{e3}| \ll 1$ (let us remind that in the quark sector $2 \times 10^{-2} \leq |V_{ub}| \leq 5 \times 10^{-2}$). We will consider the other elements of the mixing matrix as free parameters and we will see that these parameters can be determined by the two-neutrino analyses of the solar and atmospheric neutrino data. In Section 1.3.3 it will

be shown that the hypothesis $|U_{e3}| \ll 1$ can be tested in future long-baseline neutrino oscillation experiments.

If $|U_{e3}| \ll 1$, for the evolution operator in Eq.(1.33) we have the approximate expression

$$U M^2 U^\dagger + A \simeq \Delta m_{31}^2 \begin{pmatrix} \frac{A_{CG}}{\Delta m_{31}^2} & 0 & 0 \\ 0 & |U_{\mu 3}|^2 & |U_{\mu 3}||U_{\tau 3}| \\ 0 & |U_{\tau 3}||U_{\mu 3}| & |U_{\tau 3}|^2 \end{pmatrix} \quad (1.38)$$

which shows that the evolution of ν_e is decoupled from the evolution of ν_μ and ν_τ . Thus, the survival probability of atmospheric ν_e 's is equal to one and $\nu_\mu \rightarrow \nu_\tau$ transitions are independent from matter effects and are described by a two-generation formalism. In this case, the two-generation analyses of the atmospheric neutrino data in terms of $\nu_\mu \rightarrow \nu_\tau$ are appropriate in the three-neutrino scheme under consideration and yield information on the values of the parameters

$$\Delta m_{31}^2 = \Delta m_{\text{atm}}^2 \quad \text{and} \quad |U_{\mu 3}| = \sin \vartheta_{\text{atm}}. \quad (1.39)$$

1.3.2 The mixing matrix

Under the assumption $|U_{e3}| \ll 1$, the values of all the elements of the three-neutrino mixing matrix can be obtained from the results of the two-generation fits of the data of solar and atmospheric neutrino experiments. The simplest way to do it is to start from the Chau and Keung parameterization of a 3×3 mixing matrix [10, 50]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \quad (1.40)$$

where $c_{ij} \equiv \cos \vartheta_{ij}$ and $s_{ij} \equiv \sin \vartheta_{ij}$ and δ_{13} is the Dirac CP-violating phase (we have omitted three possible additional CP-violating phases in the case of Majorana neutrinos, on which there is no information).

A very small $|U_{e3}|$ implies that $|s_{13}| \ll 1$. Since the CP-violating phase δ_{13} is associated with s_{13} , it follows that CP violation is negligible in the lepton sector¹ and we have

$$U \simeq \begin{pmatrix} c_{12} & s_{12} & \ll 1 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}. \quad (1.41)$$

Using the information on $|s_{12}| \simeq |U_{e2}|$ and $|s_{23}| \simeq |U_{\mu 3}|$ given by the two-generation analyses of the results of solar and atmospheric neutrino experiments, for the moduli of the elements of the mixing matrix we obtain [46]:

$$\text{Small mixing MSW:} \quad \begin{pmatrix} \simeq 1 & 0.03 - 0.05 & \ll 1 \\ 0.02 - 0.05 & 0.71 - 0.87 & 0.49 - 0.71 \\ 0.01 - 0.04 & 0.48 - 0.71 & 0.71 - 0.87 \end{pmatrix}, \quad (1.42)$$

¹This can also be seen by noticing that the Jarlskog rephasing-invariant parameter [51–53] vanishes if one of the elements of the mixing matrix is zero.

$$\text{Large mixing MSW:} \quad \begin{pmatrix} 0.87 - 0.94 & 0.35 - 0.49 & \ll 1 \\ 0.25 - 0.43 & 0.61 - 0.82 & 0.49 - 0.71 \\ 0.17 - 0.35 & 0.42 - 0.66 & 0.71 - 0.87 \end{pmatrix}, \quad (1.43)$$

$$\text{Vacuum oscillations:} \quad \begin{pmatrix} 0.71 - 0.88 & 0.48 - 0.71 & \ll 1 \\ 0.34 - 0.61 & 0.50 - 0.76 & 0.51 - 0.71 \\ 0.24 - 0.50 & 0.36 - 0.62 & 0.71 - 0.86 \end{pmatrix}. \quad (1.44)$$

Let us remark that in the case of the small mixing angle MSW solution of the solar neutrino problem $|U_{e3}| \ll 1$ could be of the same order of magnitude as $|U_{e2}|$.

The mixing matrix (1.44) valid in the case of vacuum oscillations of solar neutrinos includes the popular bi-maximal mixing scenario that has been assumed in Refs. [54].

It is interesting to notice that, because of the large mixing of ν_μ and ν_τ with ν_2 , the transitions of solar ν_e 's in ν_μ 's and ν_τ 's are of comparable magnitude. However, this phenomenon and the values of the entries in the $(\nu_\mu, \nu_\tau) - (\nu_1, \nu_2)$ sector of the mixing matrix cannot be checked with solar neutrino experiments because the low-energy ν_μ 's and ν_τ 's coming from the sun can be detected only with neutral-current interactions, which are flavor-blind. Moreover, it will be very difficult to check the values of $|U_{\mu 1}|$, $|U_{\mu 2}|$, $|U_{\tau 1}|$ and $|U_{\tau 2}|$ in laboratory experiments because of the smallness of m_2 .

In the derivation of Eqs.(1.42)–(1.44) we have assumed that $|U_{e2}| \leq |U_{e1}|$ and $|U_{\mu 3}| \leq |U_{\tau 3}|$. The other possibilities, $|U_{e2}| \geq |U_{e1}|$ and $|U_{\mu 3}| \geq |U_{\tau 3}|$, are equivalent, respectively, to an exchange of the first and second columns and to an exchange of the second and third rows in the matrices (1.42)–(1.44). Unfortunately, these alternatives are hard to distinguish experimentally because of the above mentioned difficulty to measure directly the values of $|U_{\mu 1}|$, $|U_{\mu 2}|$, $|U_{\tau 1}|$ and $|U_{\tau 2}|$. Only the choice $|U_{e2}| \leq |U_{e1}|$, which is necessary for the MSW solutions of the solar neutrino problem, could be confirmed by the results of the new generation of solar neutrino experiments (SuperKamiokande, SNO, ICARUS, Borexino, GNO and others [55]) if they will allow to exclude the vacuum oscillation solution.

1.3.3 Accelerator long-baseline experiments

Future results from reactor long-baseline neutrino oscillation experiments (CHOOZ [44], Palo Verde [56], Kam-Land [57]) could allow to improve the upper bound (1.28) on $|U_{e3}|^2$. In this section we discuss how an improvement of this upper bound could be obtained by future accelerator long-baseline neutrino oscillation experiments that are sensitive to $\nu_\mu \rightarrow \nu_e$ transitions (K2K [58], MINOS [59], ICARUS [60] and others [61–63]).

If matter effects are not important, in the scheme under consideration the parameter $\sin^2 2\vartheta_{\mu e}$ measured in $\nu_\mu \rightarrow \nu_e$ long-baseline experiments is given by (see [48, 64])

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2|U_{\mu 3}|^2. \quad (1.45)$$

If accelerator long-baseline neutrino oscillation experiments will not observe $\nu_\mu \rightarrow \nu_e$ transitions and will place an upper bound $\sin^2 2\vartheta_{\mu e} \leq \sin^2 2\vartheta_{\mu e}^{(\max)}$, it will be possible to obtain the limit

$$|U_{e3}|^2 \leq \frac{\sin^2 2\vartheta_{\mu e}^{(\max)}}{4|U_{\mu 3}|_{(\min)}^2}, \quad (1.46)$$

where $|U_{\mu 3}|_{(\min)}^2$ is the minimum value of $|U_{\mu 3}|^2$ allowed by the solution of the atmospheric neutrino anomaly and by the observation of $\nu_\mu \rightarrow \nu_\tau$ long-baseline transitions. For example, taking $|U_{\mu 3}|_{(\min)}^2 = 0.25$ (see Eqs.(1.42)–(1.44)) we have $|U_{e3}|^2 \leq \sin^2 2\vartheta_{\mu e}^{(\max)}$. If a value of $\sin^2 2\vartheta_{\mu e}^{(\max)} \simeq 10^{-3}$, that corresponds to the sensitivity of the ICARUS experiment for one year of running [60], will be reached, it will be possible to put the upper bound $|U_{e3}| \lesssim 3 \times 10^{-2}$.

The observation of $\nu_\mu \rightarrow \nu_\tau$ transitions in long-baseline experiments will allow to establish a lower bound for $|U_{\mu 3}|^2$ because the parameter $\sin^2 2\vartheta_{\mu\tau}$ is given in the scheme under consideration by (see [48, 64])

$$\sin^2 2\vartheta_{\mu\tau} = 4|U_{\mu 3}|^2|U_{\tau 3}|^2. \quad (1.47)$$

From the unitarity relation $|U_{e3}|^2 + |U_{\mu 3}|^2 + |U_{\tau 3}|^2 = 1$ it follows that an experimental lower bound $\sin^2 2\vartheta_{\mu\tau} \geq \sin^2 2\vartheta_{\mu\tau}^{(\min)}$ allows to constraint $|U_{\mu 3}|^2$ in the range

$$\frac{1}{2} \left(1 - \sqrt{1 - \sin^2 2\vartheta_{\mu\tau}^{(\min)}} \right) \leq |U_{\mu 3}|^2 \leq \frac{1}{2} \left(1 + \sqrt{1 - \sin^2 2\vartheta_{\mu\tau}^{(\min)}} \right). \quad (1.48)$$

If $\sin^2 2\vartheta_{\mu\tau}^{(\min)}$ is found to be close to one, as suggested by the solution of the atmospheric neutrino problem, the lower bound $|U_{\mu 3}|_{(\min)}^2 = \frac{1}{2} \left(1 - \sqrt{1 - \sin^2 2\vartheta_{\mu\tau}^{(\min)}} \right)$ is close to 1/2.

If matter effects are important, the extraction of an upper bound for $|U_{e3}|^2$ from the data of $\nu_\mu \rightarrow \nu_e$ accelerator long-baseline experiments is more complicated. In this case the probability of $\nu_\mu \rightarrow \nu_e$ oscillations is given by (see [48])

$$P_{\nu_\mu \rightarrow \nu_e} = \frac{4|U_{e3}|^2|U_{\mu 3}|^2}{\left(1 - \frac{A_{CC}}{\Delta m_{31}^2}\right)^2 + 4|U_{e3}|^2 \frac{A_{CC}}{\Delta m_{31}^2}} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \sqrt{\left(1 - \frac{A_{CC}}{\Delta m_{31}^2}\right)^2 + 4|U_{e3}|^2 \frac{A_{CC}}{\Delta m_{31}^2}} \right), \quad (1.49)$$

where E is the neutrino energy and L is the distance of propagation. This probability depends on the neutrino energy not only through the explicit E in the denominator of the phase, but also through the energy dependence of $A_{CC} \equiv 2EV_{CC}$. For long-baseline neutrino beams propagating in the mantle of the Earth the charged-current effective potential $V_{CC} = \sqrt{2}G_F N_e$ is practically constant: $N_e \simeq 2 N_A \text{ cm}^{-3}$ (N_A is the Avogadro number) and $V_{CC} \simeq 1.5 \times 10^{-13} \text{ eV}$.

If long-baseline experiments will not observe $\nu_\mu \rightarrow \nu_e$ transitions (or will find that they have an extremely small probability) for neutrino energies such that $A_{CC} \lesssim \Delta m_{31}^2$, it will mean that $|U_{e3}|^2$ is small and a fit of the experimental data with the formula (1.49) will yield a stringent upper limit for $|U_{e3}|^2$ (taking into account the lower limit $|U_{\mu 3}|^2 \geq |U_{\mu 3}|_{(\min)}^2$ obtained from the solution of the atmospheric neutrino anomaly and from the observation of $\nu_\mu \rightarrow \nu_\tau$ long-baseline transitions). On the other hand, the non-observation of $\nu_\mu \rightarrow \nu_e$ transitions for neutrino energies such that $A_{CC} \gg \Delta m_{31}^2$ does not provide any information on $|U_{e3}|^2$ because in this case the transition probability (1.49) is suppressed by the matter effect. Hence, in order to check the hypothesis $|U_{e3}| \ll 1$, as well as to have some possibility to observe $\nu_\mu \rightarrow \nu_e$ transitions if this hypothesis is wrong, it is necessary that a substantial part of the energy spectrum of the neutrino beam lies

below

$$\frac{\Delta m_{31}^2}{2V_{CC}} \simeq 30 \text{ GeV} \left(\frac{\Delta m_{31}^2}{10^{-2} \text{ eV}^2} \right). \quad (1.50)$$

This requirement will be satisfied in the accelerator long-baseline experiments under preparation (K2K [58], MINOS [59], ICARUS [60] and others [61–63]) if Δm_{31}^2 is not much smaller than 10^{-2} eV^2 .

1.3.4 Neutrinoless double- β decay

Let us consider now the implications of the result in Eq.(1.28) for neutrinoless double- β decay experiments. The investigation of neutrino oscillations does not allow (see [6, 7] to answer the fundamental question: are massive neutrinos Dirac or Majorana particles? Only investigations of neutrinoless double- β decay could allow to answer this question. In the case of a three-neutrino mass hierarchy for the effective Majorana mass we have [65,66]

$$|\langle m \rangle| \simeq |U_{e3}|^2 \sqrt{\Delta m_{31}^2}. \quad (1.51)$$

Taking into account the bound (1.28) on $|U_{e3}|^2$, we obtain the following constraint for the effective Majorana mass in neutrinoless double- β decay [66]:

$$|\langle m \rangle| \lesssim a_e^0 \sqrt{\Delta m_{31}^2}. \quad (1.52)$$

The value of this upper bound as a function Δm_{31}^2 obtained from 90% CL exclusion plots of the Bugey [29] and CHOOZ [44] experiments is presented in Fig. 1.3 (the solid and dashed line, respectively). The region on the right of the thick straight solid line is forbidden by the unitarity bound $|\langle m \rangle| \leq \sqrt{\Delta m_{31}^2}$.

Also the results of the Super-Kamiokande atmospheric neutrino experiment [18] imply an upper bound for $|U_{e3}|^2$. The shadowed region in Fig. 1.3 shows the region allowed by Super-Kamiokande results at 90% CL that we have obtained using the results of three-neutrino analysis performed by Yasuda [67].

Figure 1.3 shows that the results of the Super-Kamiokande and CHOOZ experiments imply that $|\langle m \rangle| \lesssim 10^{-2} \text{ eV}$.

The observation of neutrinoless double- β decay with a probability that corresponds to a value of $|\langle m \rangle|$ significantly larger than 10^{-2} eV would mean that the masses of three neutrinos do not have a hierarchical pattern and/or exotic mechanisms (right-handed currents, supersymmetry with violation of R-parity, ... [68,69]) are responsible for the process.

Let us notice that from the results of the Heidelberg-Moscow ^{76}Ge experiment [70] it follows that $|\langle m \rangle| \lesssim 0.5 - 1.5 \text{ eV}$. The next generation [71] of neutrinoless double- β experiments will reach $|\langle m \rangle| \simeq 10^{-1} \text{ eV}$. Possibilities to reach $|\langle m \rangle| \simeq 10^{-2} \text{ eV}$ are under discussion [71].

1.4 Discussion

Xing: A hierarchy of Δm^2 's does not imply a hierarchy of masses.

Bilenky: Yes, I agree. However, on the basis of a similarity with the observed hierarchies of the masses of quarks and charged leptons, a hierarchy of Δm^2 's is a strong hint in favor of a hierarchy of neutrino masses.

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2 A review of recent neutrino oscillation solutions and their implications for the future experimental neutrino program — Achim Geiser

2.1 Introduction

Currently there is a wealth of unexplained phenomena in neutrino physics which can be interpreted as indications for the existence of neutrino oscillations. The solar neutrino problem [1], lacking a satisfactory astrophysical solution, is commonly attributed to the disappearance of electron neutrinos into some other neutrino type. In the wake of the recent Super-Kamiokande results [2], the atmospheric neutrino anomaly [3] is interpreted as evidence for neutrino oscillations involving ν_μ disappearance. The LSND experiment [4] claims direct evidence for $\nu_\mu - \nu_e$ oscillations in a region which is partially unconstrained by other experiments. Furthermore, neutrinos are a prime candidate for a partial solution to the missing dark matter problem [5] if at least one mass eigenstate lies in the eV range. A more detailed discussion of the experimental indications will be presented elsewhere.

It is the purpose of this contribution to systematically review the different classes of possible solutions and discuss some of their features, including their predictions for future experiments. An attempt is made to include all oscillation scenarios published after 1995, and relevant preprints not older than one year. First, all these scenarios will be classified according to general criteria. Then, some of these models will be discussed in more detail.

2.2 Classification of neutrino oscillation solutions

Results of neutrino oscillation experiments are often expressed in terms of an effective two flavour oscillation scheme with a mixing angle $\sin^2 2\theta$ between the two flavours and a mass difference δm^2 between the two relevant mass eigenstates. The indications for neutrino oscillations from solar neutrinos, atmospheric neutrinos and LSND each suggest a different value of δm^2 . Unfortunately it is not possible to find a unique solution simultaneously satisfying all these indications.

Solutions involving the 3 known active neutrino flavours yield only two independent δm^2 values. The problem is therefore overconstrained, and some experimental evidence has to be discarded (or equivalently, a bad fit has to be accepted) in order to find a solution. This induces some arbitrariness in which parts of the data are deemed to be reliable, and which parts should be ignored. Once some information has been discarded, the remaining information is often not sufficient to uniquely constrain the 3 mixing angles.

Solutions involving 1 or more sterile neutrinos are generally underconstrained due to the 6 or more mixing angles involved. Therefore many of the models of this kind make some simplifying assumptions which reduce the parameter space before attempting to find a solution, or impose constraints obtained from specific GUT models or other extensions of the standard model.

In general, there are four big classes of possible solutions

- Solutions trying to accommodate all the experimental evidence in the standard 3 neutrino scheme, at the expense of accepting a bad fit to part of the data [6–11].
- Solutions which simply discard one of the experimental indications [12–23]. A good fit to the remaining indications can then be obtained from three active neutrinos. The current “standard” solution (dropping LSND) falls into this class.
- Solutions which invoke 1 additional light sterile neutrino. This is the minimum required to accommodate the three different δm^2 values discussed above. Commonly the sterile neutrino is used to solve the solar neutrino problem [24–27], but scenarios explaining the atmospheric neutrino anomaly through active-sterile oscillations are equally possible [27–29]. Alternatively, the sterile neutrino can be used to explain LSND [30]. Many models predicting such scenarios have been investigated [31].
- Solutions which invoke more than one sterile neutrino (usually 3, motivated by assuming some symmetry between the active and sterile neutrinos) [32–36]. It has recently been shown that Big Bang nucleosynthesis limits which seemed to exclude some of these scenarios can be evaded [37].

Other solutions can be derived by partially replacing the oscillation hypothesis by other non-standard model effects (neutrino magnetic moments, anomalous interactions [38], ...) or by discarding more of the experimental evidence than the minimum needed to satisfy a given scenario.

In order to put some structure into the discussion of the multitude of oscillation scenarios, we adopt a formal classification of each solution according to its interpretation of the various pieces of experimental evidence.

The classification criteria used are

A. The solar neutrino problem is

- I. not caused by neutrino oscillations (astrophysical solution, spin flip due to neutrino magnetic moment, ...)
- II. due to $\nu_e \rightarrow \nu_\mu$ (or ν_τ) oscillations
- III. due to $\nu_e \rightarrow \nu_S$ oscillations

B. The atmospheric neutrino anomaly is

- I. not caused by neutrino oscillations (anomalous ν_τ interactions, ...)
- II. due to $\nu_\mu \rightarrow \nu_\tau$ oscillations
- III. due to a linear combination of $\nu_\mu - \nu_e$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations
- IV. due to $\nu_\mu \rightarrow \nu_S$ oscillations

C. The LSND result is

- I. not caused by neutrino oscillations
- II. due to direct $\nu_\mu \rightarrow \nu_e$ oscillations (effective 2x2 mixing matrix)
- III. due to indirect $\nu_\mu \rightarrow \nu_e$ oscillations (full 3x3 matrix is relevant)

In addition, all scenarios can be classified in terms of the different MSW and vacuum oscillation schemes used to solve the solar neutrino problem [1]. Further classification criteria could be the choice of the relevant δm^2 , the level of compatibility with existing reactor and accelerator limits and limits on double beta decay, the compatibility with different dark matter scenarios, and the compatibility with Big Bang nucleosynthesis and supernova processes. An explicit formal consideration of these criteria would however be impractical to handle. They will therefore be discussed elsewhere.

Table 2.1 classifies all recent neutrino oscillation solutions according to categories A.-C., and lists the corresponding expectations for current and future neutrino oscillation experiments. It is interesting to note that there is almost no combination of elements of this classification scheme which has NOT been discussed in the recent literature as a possible scenario. Conversely, there are only very few linear combinations of results of future measurements which would NOT correspond to a possible oscillation scenario.

2.3 Discussion of selected models

2.3.1 Three neutrino solutions including LSND

Before the recent results from CHOOZ [43] and Super-Kamiokande [2], this used to be one of the favorite options. Discarding the solar neutrino deficit, one can easily build models including LSND and atmospheric neutrinos only [14, 25]. However, currently no serious alternative to the neutrino oscillation hypothesis of the solar neutrino deficit exists. On the other hand, one could discard the atmospheric neutrino anomaly and build models using LSND and solar neutrinos only [13]. Going one step less far, the model of Cardall

Table 2.1: Classification of neutrino oscillation solutions in terms of their interpretation of the current experimental data. The predictions for selected ongoing and future experiments are given in each case. The roman numbers refer to the classification scheme described in the text. The symbol + stands for the expectation of a positive effect (deviation from the no oscillation expectation) and implies that the corresponding scenario is excluded if no signal is observed. The symbol - indicates the expectation of a negative result and implies that the scenario is excluded if a positive signal is seen. The symbol 0 indicates that both positive and negative results are possible in a given scenario, often depending on the results of other future measurements. In many cases it means that the scenario is favoured in the case of a positive effect, but not (completely) excluded if no signal is seen. The experiments are explained in section 2.5.

s o l a r	a t m o s p h e r i c	L S N D	type of model + references	C H O R U S / N O M A D	s b h a o s r e g p t l p / i b e n e s r e a r m a n e e	l τ o i n a a s e r l n i c n e e	l d o i n s g a p b p a e s a e r l a i n c e e	S u p e r K . N C / C C	C H O R U S / P a l o V .	K a m l a n d	S N O N C / C C	B o r e x i n o / I o d .	B o o n e / P S I 2 1 6	
A.	B.	C.												
I	I	I	no oscillations	-	-	-	-	-	-	-	-	-	-	-
I	I	II/III	LSND only [4, 15, 70]	0/-	0/+	0/+	-	-	-	-	-	-	-	+
I	II	I	atm. ν_τ only [2, 39]	-	-	0	0	+	-	-	-	-	-	-
I	II	II	atm. ν_τ + LSND [14, 25]	-	-	0	0	+	-	-	-	-	-	+
I	III	I	atm. $\nu_\tau + \nu_e$ only [19]	-	-	0	0	+	0	0	-	-	-	-
I	III	II/III	atm. $\nu_\tau + \nu_e$ + LSND [7, 14, 25]	-	-/+	0/+	0	+	0	0	-	-	-	+
I	IV	I	atm. ν_s only [39]	0	0	0	0	-	0	-	-	-	-	-
I	IV	II/III	atm. ν_s + LSND [40]	0/-	0/+	0/+	0	-	0	-	-	-	-	+
II	I	I	solar $\nu_{\mu/\tau}$ only [1]	0	0	0	-	-	-	0	+	0	-	-
II	I	II/III	solar $\nu_{\mu/\tau}$ + LSND [13]	0/-	0/+	0/+	-	-	-	0	+	0	+	-
II	II	I	“standard” 3 ν [12, 17, 22, 23]	-	-	0	0	+	-	0	+	0	-	-
II	II	III	Cardall/Fuller [6, 7]	-	+	+	0	+	-	0	+	0	+	-
II	III	I	Harr./P./Sc. [16]+oth. [10, 19, 22]	-	-	+	0	+	0	0	+	0	-	-
II	III	III	Ack./Pakv. [8]+oth. [9–11]	-	+	+	0	+	0	0	+	0	+	-
II	IV	I	4 ν , ν_μ - ν_s mix. [27–29]	0	0	0	0	-	0	0	+	0	-	-
II	IV	II/III	4 ν , ν_μ - ν_s +LSND [27–29]	0/-	0/+	0/+	0	-	0	0	+	0	+	-
III	I	I	solar ν_s only [1]	0	0	0	-	-	-	-	-	0	-	-
III	I	II/III	solar ν_s + LSND [41]	0/-	0/+	0/+	-	-	-	-	-	0	+	-
III	II	I	solar ν_s + atm. ν_τ [42]	-	-	0	0	+	-	-	-	0	-	-
III	II	II	“standard” 4 ν [24, 25, 32]	-	-	0	0	+	-	-	-	0	+	-
III	III	I	solar ν_s + atm. $\nu_\tau + \nu_e$ [42]	-	-	0	0	+	0	0	-	0	-	-
III	III	II/III	idem + LSND [26, 27]	-	-/+	0/+	0	+	0	0	-	0	+	-
III	IV	I	sol.+atm. ν_s [28, 33–35]	0	0	0	0	-	0	-	-	0	-	-
III	IV	II	sol.+atm. ν_s +LSND [28, 33, 34]	0/-	0/+	0/+	0	-	0	-	-	0	+	-

and Fuller [6] tries to reconcile LSND, atmospheric, and solar neutrinos by ignoring the energy dependence of the up-down asymmetry in the Kamiokande Multi-GeV data [44]. The same δm^2 can then be used for LSND and atmospheric neutrinos. This scenario is now strongly disfavored by the much more significant up/down asymmetry observed by Super-Kamiokande [2]. An alternative is to merge the δm^2 of the atmospheric and solar neutrinos by discarding the Homestake [45] result or by somewhat increasing its error [46]. This allows an energy-independent suppression of the solar neutrinos at high δm^2 . However, the corresponding model of Acker and Pakvasa [8] is not consistent with the recent CHOOZ and Super-Kamiokande results, which strongly constrain the possible $\nu_\mu - \nu_e$ contribution [20]. A new model by Thun and McKee [9] merges the Cardall/Fuller and Acker/Pakvasa schemes by letting both the solar neutrino and the LSND δm^2 contribute significantly to the atmospheric neutrino anomaly. This model can successfully describe the Super-Kamiokande up/down asymmetry of the e/μ ratio, but fails to fit the shape of the individual ν_μ and ν_e fluxes and asymmetries. It is therefore excluded unless large unknown systematic effects are assumed (i.e. the corresponding flux measurements systematically disagree [47]). On the other hand, this kind of model might be interesting in the context of a recent claim that the KARMEN anomaly [48] can be explained through neutrino oscillations [49].

2.3.2 Three neutrino solutions excluding LSND

One currently favoured way out of the dilemma is to discard the LSND result [4], which has so far not been confirmed by KARMEN [50] or other experiments [51]. This leads to a scenario which is very close to one of the standard scenarios before the LSND claim [12]. The atmospheric neutrino anomaly is interpreted in terms of $\nu_\mu - \nu_\tau$ oscillations, while the solar neutrino deficit is ascribed to $\nu_\mu - \nu_e$ oscillations. Both MSW and vacuum solutions are allowed in this context [1]. Variations of this scheme allow an additional significant $\nu_\mu - \nu_e$ contribution to the atmospheric neutrino anomaly [17,19]. In particular, the three-fold maximal mixing scenario [16] often cited as model of Harrison, Perkins, and Scott is not completely excluded by CHOOZ and Super-Kamiokande [19]. However, the pure $\nu_\mu - \nu_e$ interpretation of the atmospheric neutrino data is no longer allowed [20]. Models explaining both the solar and atmospheric neutrino problems through $\nu_\mu - \nu_e$ oscillations [21] are therefore excluded. Motivations why neutrino mixing angles could/should be large are given by various authors [22,23].

2.4 Sterile neutrinos

If the current indications for three independent δm^2 are confirmed, the only way out is the introduction of at least a fourth neutrino. LEP [52] has measured the number of neutrinos coupling to weak interactions to be $N_\nu = 2.994 \pm 0.012$, corresponding to the three known lepton generations. Any extra neutrinos must therefore be either very massive ($m_\nu > M_Z/2$) or sterile with respect to weak interactions. In a minimal extension of the standard model, these could e.g. be the right-handed (left-handed) partners of the standard model neutrino (antineutrino). A tiny lepton-number violating interaction could then induce neutrino-antineutrino oscillations, in analogy to $K^0 - \bar{K}^0$ oscillations [53]. Since oscillations do not change the spin orientation, left-handed neutrinos would oscillate

into left-handed antineutrinos which appear sterile. Alternatively, extra neutrino singlets or multiplets can be introduced in the context of Grand Unified Theories (see e.g. [29] and references therein) or other extensions of the standard model [33]. Many of these schemes invoke the see-saw mechanism [54], which makes some of the additional neutrinos too heavy to be detected. Current experiments can not clearly distinguish oscillations with sterile neutrinos from standard flavour oscillations for either atmospheric [39] or solar [1] neutrinos. $\nu - \nu_s$ oscillations are therefore allowed in both cases. Finally, it is worth noting that many of the models advocating light sterile neutrinos are justified without the explicit requirement of the LSND constraint, and hence do not depend on LSND being confirmed.

2.4.1 Four neutrino solutions

Many models consider one sterile neutrino in addition to the active ones. This is the minimum required to obtain three independent δm^2 's, and is sometimes motivated by specific GUT models. Two classes of such models can be distinguished, depending on which of the experimental observations is ascribed to active/sterile oscillations. Schemes invoking $\nu_e - \nu_s$ oscillations for the solar neutrino deficit [24, 25] are the oldest class of such models. In this context, the atmospheric neutrino deficit is explained by $\nu_\mu - \nu_\tau$ oscillations, and LSND by $\nu_\mu - \nu_e$ oscillations. More complicated variants of these models are possible [26, 27]. Alternatively, $\nu_\mu - \nu_s$ oscillations can be invoked to describe the atmospheric neutrino anomaly [27–29]. In this case, the solar neutrino deficit can be some linear combination of $\nu_e - \nu_\mu$, $\nu_e - \nu_\tau$, and $\nu_e - \nu_s$ oscillations. The possibility that the LSND signal is due to indirect oscillations involving the ν_s has also been considered [30]. Mass textures for all these solutions are being investigated [31].

2.4.2 Five or six neutrino solutions

Models with more than four neutrinos have been unpopular due to their apparent conflict with bounds on the number of neutrinos from Big Bang nucleosynthesis [55, 56]. However, it has recently been demonstrated that these limits can be evaded through modifications of the Big Bang nucleosynthesis model which are a consequence of the active/sterile neutrino oscillations themselves [37]. Scenarios with five light neutrinos are considered in [32]. A more natural configuration of 6 neutrinos can be obtained by assigning a light sterile (right-handed) neutrino to each lepton generation through some symmetry (e.g. parity). In the context of such scenarios [33, 34], both atmospheric and solar neutrino oscillations are explained through close to maximal active/sterile oscillations, while the LSND result is interpreted as standard $\nu_\mu - \nu_e$ oscillations. Other phenomenologically similar models [36] invoke an unstable ν_τ in the MeV mass range to evade the Big Bang nucleosynthesis constraint.

2.5 Implications for future experiments

As can be inferred from table 2.1 almost any linear combination of the outcome of ongoing and future experiments corresponds to a possible oscillation scenario. The predictions for these experiments therefore depend on the experimental and theoretical bias of which current experimental results are believed to be true and how they are interpreted.

No single experiment can uniquely *confirm* any of the discussed scenarios, but certain combinations of future experimental results will rule out or support some of them. (Mini)-Boone [57] and/or PS I216 [58], if approved, will directly check the LSND claim, therefore ruling out half of the listed scenarios. Similarly, the measurement of the NC/CC ratio at the Sudbury Neutrino Observatory (SNO [59]) will uniquely check the $\nu_e - \nu_\mu$ (or $\nu_e - \nu_\tau$) interpretation of the solar neutrino deficit. Measurements of the NC/CC ratio in atmospheric neutrinos, for instance by Super-Kamiokande [60] are very powerful in principle for the distinction of atmospheric $\nu_\mu - \nu_{\tau/e}$ and $\nu_\mu - \nu_s$ oscillations. In practice, they might be limited by low statistics and/or large systematic errors [61]. Alternatively, the combination of future long [62, 63] and short [64] or intermediate [65] baseline τ appearance experiments (but not the long baseline experiments alone [61]) can achieve the same distinction with much higher significance, provided the atmospheric δm^2 is not too small. With the same caveat, long baseline disappearance experiments [63, 66, 67] will check the atmospheric neutrino anomaly, and appearance experiments [62, 63, 67] will test any significant $\nu_\mu - \nu_e$ contribution. The observation of an oscillation signal beyond the current limits at the CHOOZ [43] or Palo Verde [68] reactors would establish such a contribution. The observation of $\nu_\mu - \nu_\tau$ oscillations in ongoing (CHORUS/NOMAD [69]) or future [64, 65] short or intermediate baseline experiments could force the $\nu_\mu - \nu_s$ interpretation of the atmospheric neutrino anomaly. Alternatively, it could confirm the oscillation scenario II for LSND [70] invoked in the Cardall/Fuller, Acker/Pakvasa, and Thun/McKee schemes [71]. The KamLand experiment [72] will check the large angle MSW solution of the solar neutrino problem [1], while the Iodine [73] and Borexino [74] experiments will try to confirm the energy dependence of the solar neutrino deficit.

2.6 Conclusions

An attempt has been made to classify all recently proposed neutrino oscillation solutions according to their interpretation of the current experimental evidence, and to discuss their implications for future experiments. At present, no single scenario is emerging as “the” obvious candidate. Essentially all planned future neutrino experiments are needed and useful to constrain and/or exclude certain classes of scenarios, and almost any combination of future experimental results corresponds to a possible oscillation scheme.

2.7 Discussion

Fogli: A detailed quantitative calculation shows that the model of Thun and McKee (as well as the other models in reference [9]) gives a very bad fit of the separate zenith angle distributions of atmospheric e -like and μ -like events measured by Super-Kamiokande (see Figs 2.1 and 2.2). The model of Thun and McKee assumes that the atmospheric neutrino anomaly is explained by dominant $\nu_\mu \rightarrow \nu_e$ transitions with $\Delta m_{21}^2 \lesssim 10^{-3} \text{ eV}^2$, *i.e.* below the CHOOZ bound [43]. Sub-dominant $\nu_\mu \rightarrow \nu_\tau$ oscillations due to $\Delta m_{23}^2 \simeq 0.4 \text{ eV}^2$ cause an additional energy-independent suppression of the atmospheric muon neutrino flux.

As one can see from Fig. 2.1 (obtained with the method described in the second paper of Ref. [12]), the resulting distribution of μ -like events in the Super-Kamiokande

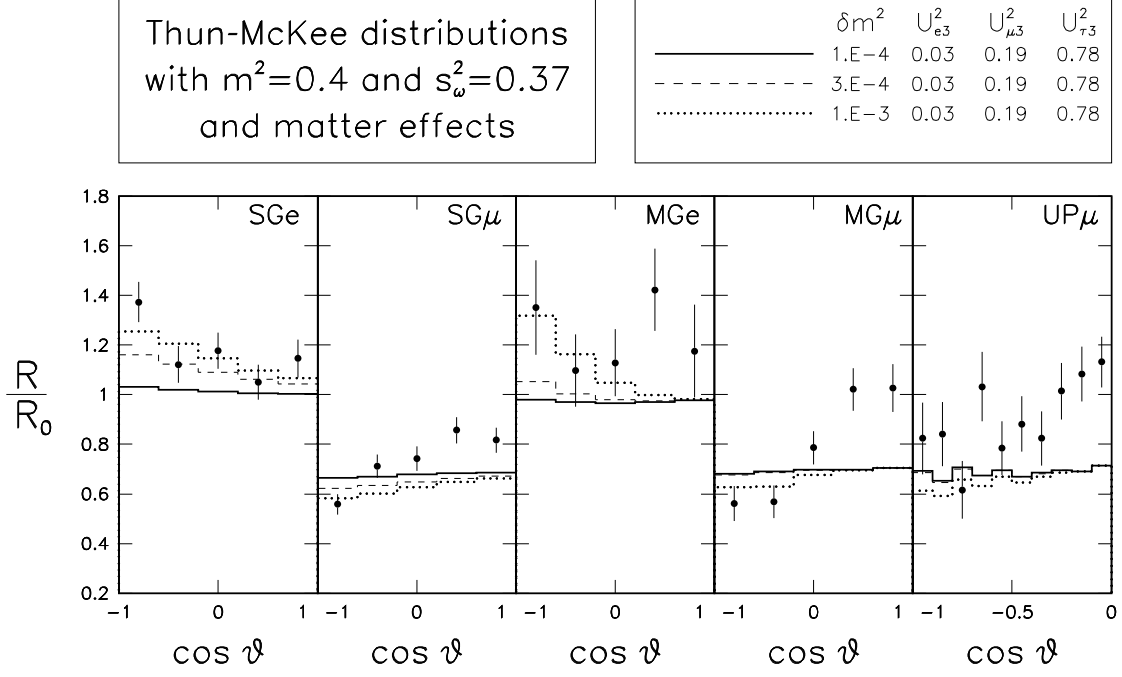


Figure 2.1

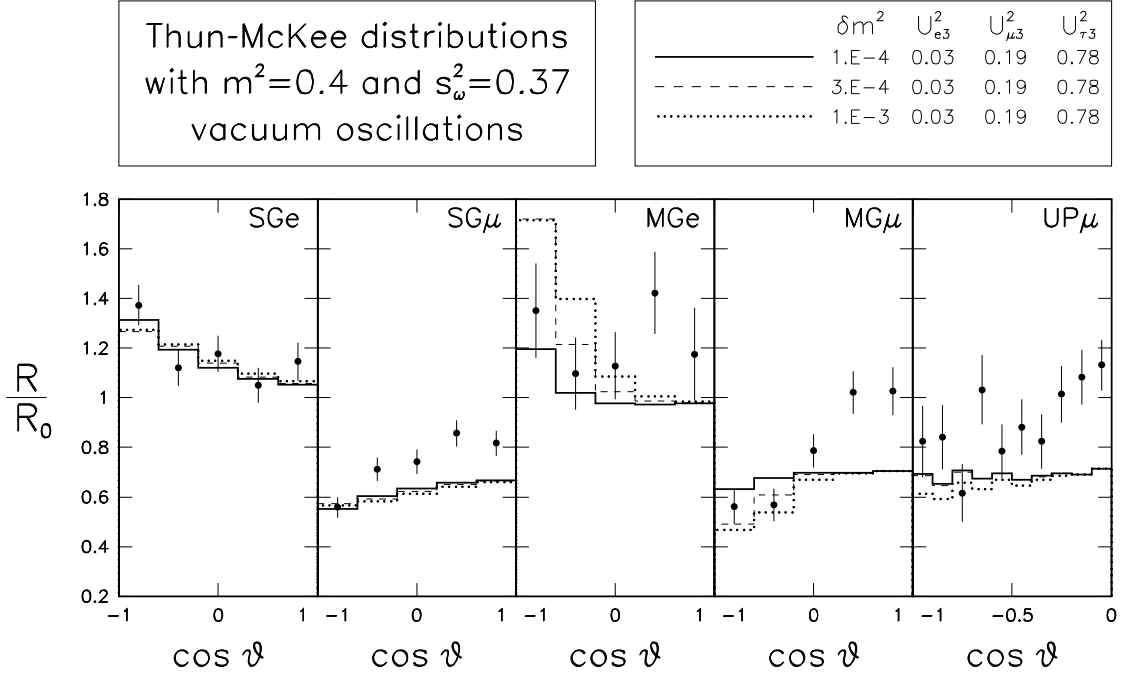


Figure 2.2

detector do not fit the data because they are too flat. The reasons for this flatness are:

1. The matter effect suppresses the effective mixing. From the comparison of Figs 2.1 and 2.2 one can see that the matter effect is not negligible (a detailed

calculation shows that the matter effect can give up to 60% change of the μ/e ratio of upward-going events).

2. The presence of the sub-dominant energy-independent $\nu_\mu \rightarrow \nu_\tau$ oscillations.

Furthermore, one must take into account that the model of Thun and McKee gives a bad fit of solar neutrino data and a marginal fit of reactor and accelerator neutrino oscillation data.

Geiser: I verified that it gives a bad fit of the separate e -like and μ -like Super-Kamiokande data as a function of L/E . However, this incompatibility might be attenuated by changing the spectral slope of the cosmic rays used for the theoretical calculations of the atmospheric neutrino fluxes. (note added: after studying figs. 2.1 and 2.2 in more detail, I agree that it seems indeed hard to account for the Multi-GeV muon up-down asymmetry once matter effects are taken into account.)

Frekers: Which are the astrophysical constraints on the scheme with six pseudo-Dirac neutrinos? In particular, are there constraints from Big-Bang nucleosynthesis and supernovae? What about the energy loss in supernovae?

Geiser: The six pseudo-Dirac neutrino scheme [34] is indeed compatible with Big Bang nucleosynthesis through the mechanism proposed by Foot and Volkas (Ref. [37]). I did not study the compatibility with supernova processes in detail, but I am not aware of any incompatibility with cases treated in the literature so far.

Langacker: Maximal mixing oscillations of active into sterile neutrinos in the supernova core may cause a too large energy loss.

Giunti: The maximal mixing oscillations in the supernova core of active into sterile neutrinos is suppressed if the effective potential is bigger than all relevant Δm^2 . In this case the effective mixing angles are close to $\pi/2$, *i.e.* in practice there is no mixing. However, transitions of active into sterile neutrinos can occur while the neutrinos escape the supernova and should be studied in detail.

Sarkar: Big Bang nucleosynthesis is well studied and certainly rules out 3 additional maximally mixed sterile neutrinos. With regard to evading this bound through the Foot-Volkas mechanism [37], please note that the sign of the lepton asymmetry generated is arbitrary so the value of N_ν can go up as well as down!

Frekers: People proposing new models should explain clearly their compatibility with physical and astrophysical data.

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3 Detection of $\nu_\mu \rightarrow \nu_e$ at $\Delta m^2 \gtrsim 10^{-3} \text{ eV}^2$ in ICARUS — Stan Otwinowski

The recent results of the Super-Kamiokande experiment appear to provide strong evidence for $\nu_\mu \rightarrow \nu_x$ atmospheric neutrino oscillations [1]. The most interesting possibility is that the ν_x is a ν_τ . A large effort at FNAL and the CERN-Gran Sasso Laboratories will be directed at the detection of a τ appearance. However if the relevant Δm^2 is less than 10^{-3} eV^2 this is a very difficult task since the τ threshold energy of 3.5 GeV forces the use of relatively high energy neutrinos. It is conceivable that the Super-Kamiokande results could be confirmed without determining the actual nature of the ν_x (K2K or MINOS or ICARUS) leaving open the possibility that ν_x is a sterile neutrino. There is another way to have some hint that ν_x is an active neutrino by exploiting the possibility of three neutrino flavour mixing. The basic idea is that at the same level if the Super-Kamiokande results are due to $\nu_\mu \rightarrow \nu_\tau$ then the processes $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ may occur at the same Δm^2 . Observation of the $\nu_\mu \rightarrow \nu_e$ process at foreseen electron energy region does not rule out the sterile neutrino but, we believe, the sterile neutrino hypothesis would be less favoured.

We believe that ICARUS [2] is the ideal detector for this observation, since the excellent vertex resolution can be used to discriminate against neutral current π^0 events.

There are two possibilities for this detection:

1. $\nu_\mu \rightarrow \nu_e$ has a transition probability bigger than 10% (up to the CHOOZ limit [3]). In this case the signal would be much larger than the ν_e background in the LBL beam (0.8%) and detection is likely. It seems very possible that this will occur since the large $\nu_\mu \rightarrow \nu_\tau$ transition probability suggests that the neutrino sector is unlike the quark mixing sector with the small CKM mixing angles — all of the mixing angles could be relatively large.
2. The transition probability is less than 10^{-1} (say 5×10^{-2}). It may still be possible to detect the $\nu_\mu \rightarrow \nu_e$ transitions through the observation of a modification of the ν_e energy spectrum. This is due to the fact that the normal ν_e beam is somewhat harder than the ν_μ beam because of the ν_e origin from K^\pm decays and the very small forward angles accepted by the detector in the LBL beam. For example, for certain values of the mixing parameters we expect considerable $\nu_\mu \rightarrow \nu_e$ transitions at 5–10 GeV neutrino energy — at lower energy than the normal ν_e beam (Fig. 3.1). A lower energy ν_μ beam, as recently suggested by A. Ereditato in Ref. [4], would be even more useful (see Fig. 3.1).

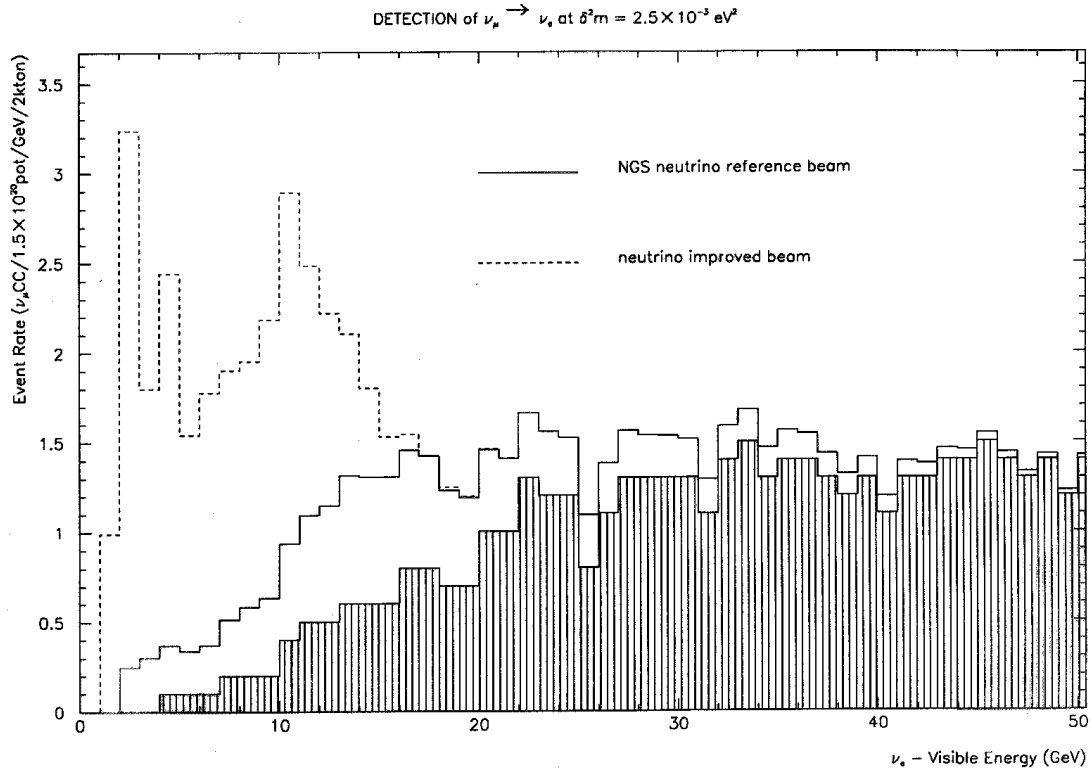


Figure 3.1

As an example we consider a model of Cardall, Fuller and Cline [5], where a 3- ν analysis was studied. In the case of the mixing matrix

$$U_{\text{CF,lma}} \approx \begin{pmatrix} 0.873 & 0.478 & 0.100 \\ -0.330 & 0.428 & 0.841 \\ 0.359 & -0.767 & 0.532 \end{pmatrix} \quad (3.1)$$

we have

$$\nu_\mu \rightarrow \nu_\tau \quad P \approx 1, \quad (3.2)$$

$$\nu_\mu \rightarrow \nu_e \quad P \approx 10^{-1}. \quad (3.3)$$

This model also fits the Solar neutrino data with the large mixing angle (lma) solution at $\Delta m^2 \approx 2 \times 10^{-5} \text{ eV}^2$. This is only an example and a more detailed analysis of the 3- ν mixing is called for (see Ref. [5]).

3.1 Discussion

Langacker: I think that the observation of long-baseline $\nu_\mu \rightarrow \nu_e$ oscillations cannot help to distinguish the $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_s$ solutions of the atmospheric neutrino anomaly.

Giunti: I also think so. For example, with four neutrinos there can be simultaneous large $\nu_\mu \rightarrow \nu_s$ oscillations and observable $\nu_\mu \rightarrow \nu_e$ oscillations at the atmospheric scale of Δm^2 . However, in the case of four neutrinos large long-baseline $\nu_\mu \rightarrow \nu_e$ oscillations are incompatible with the LSND indication in favor of short-baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ oscillations [6].

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4 Phenomenological Ansätze for Large Mixing of Three Neutrinos — Zhi-Zhong Xing

4.1 Introduction

Recently the Super-Kamiokande Collaboration has reported new and stronger evidence for the existence of the atmospheric neutrino anomaly. The data particularly favor an interpretation of the observed muon-neutrino deficit by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with the mass-squared difference $\Delta m_{\text{atm}}^2 \approx (0.5 - 6) \times 10^{-3} \text{ eV}^2$ and the mixing factor $\sin^2 2\theta_{\text{atm}} > 0.82$ at the 90% confidence level [1]. The long-standing solar neutrino deficit has also been confirmed in the Super-Kamiokande experiment. Analyses of the energy shape and day-night spectra of solar neutrinos seem to favor the “Just-So” mechanism, i.e., the long-wavelength vacuum oscillation with $\Delta m_{\text{sun}}^2 \approx 10^{-10} \text{ eV}^2$ and $\sin^2 2\theta_{\text{sun}} \approx 1$ [2]. However, it remains too early to justify whether the solar neutrino anomaly is attributed to the “Just-So” oscillation or to the matter-enhanced oscillation (i.e., the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism). An analysis of all available solar neutrino data based on the latter gives two different ranges of the oscillation parameters [3]: the large-angle MSW solution with $\Delta m_{\text{sun}}^2 \approx 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta_{\text{sun}} \approx 0.8$ [4] or the small-angle MSW solution with $\Delta m_{\text{sun}}^2 \approx 5 \times 10^{-6} \text{ eV}^2$ and $\sin^2 2\theta_{\text{sun}} \approx 5 \times 10^{-3}$. If the large mixing angles θ_{atm} and θ_{sun} in the “Just-So” or the large-angle MSW scenario are finally confirmed, one would have an indication that the physics responsible for neutrino masses and lepton flavor mixing might be qualitatively different from that for the quark sector.

The small mixing angle θ_{sun} appearing in the small-angle MSW scenario, on the other hand, seems similar to the small angles observed in the quark flavor mixing phenomenon.

The LSND evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations [5], whose parameters lie in the ranges $\Delta m_{\text{LSND}}^2 \approx (0.4 - 2) \text{ eV}^2$ and $\sin^2 2\theta_{\text{LSND}} \approx 10^{-3} - 10^{-2}$, was not confirmed by the recent KARMEN experiment [6]. Because a further examination of the LSND result will be available in the coming years, the conservative approach is to put it aside tentatively from the evidence of solar and atmospheric neutrino oscillations. Indeed it is extremely difficult, if not impossible, to accommodate all solar, atmospheric and LSND oscillation data within the scheme of three active neutrinos.

It is remarkable that the CHOOZ experiment [7], in which the survival probability of $\bar{\nu}_e$ neutrino was measured, indicates that $\sin^2 2\theta_{\text{CH}} < 0.18$ if $\Delta m_{\text{CH}}^2 \geq 9 \times 10^{-4} \text{ eV}^2$. This result supports that from the Super-Kamiokande experiment, i.e., the atmospheric neutrino deficit comes most likely from $\nu_\mu \leftrightarrow \nu_\tau$ oscillations instead of $\nu_\mu \leftrightarrow \nu_e$ oscillations in the three-neutrino framework. In particular, it turns out that the (1,3) element of the 3×3 lepton flavor mixing matrix is naturally small in magnitude and the atmospheric and solar neutrino oscillations approximately decouple – they are separately dominated by a single mass scale (see, e.g., Refs. [8, 9]):

$$\begin{aligned} \nu_\mu \leftrightarrow \nu_e \text{ oscillation : } \Delta m_{\text{sun}}^2 &= \Delta m_{21}^2, \\ \nu_\mu \leftrightarrow \nu_\tau \text{ oscillation : } \Delta m_{\text{atm}}^2 &= \Delta m_{32}^2 \approx \Delta m_{31}^2, \end{aligned} \quad (4.1)$$

where $\Delta m_{ij}^2 \equiv |m_i^2 - m_j^2|$ and m_i (for $i = 1, 2, 3$) is the neutrino mass. The smallness of the (1,3) mixing element in this simple picture, compared with the smallness of the (1,3) element in the 3×3 quark mixing matrix [10], reflects some kind of similarity between lepton and quark flavor mixings. Whether our present understanding of solar and atmospheric neutrino oscillations as described in Eq. (1) is close to the truth or not, however, remains an open question.

To interpret current data on atmospheric and solar neutrino oscillations, one may follow a phenomenological way to construct the lepton flavor mixing matrix which can account for the experimentally favored values of θ_{atm} and θ_{sun} . Any insight into the dynamics of lepton mass generation, however, requires nontrivial theoretical steps beyond the standard model [11]. Note that the oscillation parameters Δm^2 and $\sin^2 2\theta$ are in general expected to correlate with each other, as the latter comes from the mismatch between charged lepton and neutrino mass matrices and should depend on the ratio(s) of neutrino masses. One can therefore distinguish between two different types of models, in which $\sin^2 2\theta$ rely specifically on Δm^2 , or in which $\sin^2 2\theta$ is independent of any neutrino mass. In all experimental analyses, of course, Δm^2 and $\sin^2 2\theta$ are treated as two independent parameters.

Before a successful theory of lepton mass generation and flavor mixing appears, the proper phenomenological approach might be first to identify the patterns of lepton mass matrices guided by some kinds of possible symmetries, and then to calculate the lepton flavor mixing matrix. The “success” of such lepton mass and mixing ansätze can only be “justified” by their consequences, i.e., if they agree well with current data on neutrino oscillations and if they are consistent with some other constraints on neutrinos (e.g., the constraints from the neutrinoless $\beta\beta$ -decay or from the hot dark matter). We hope that our phenomenological attempts can provide useful hints towards the theoretical solution of lepton mass and mixing problems.

Following the strategy outlined above we are going to discuss a simple phenomenological model of lepton masses and flavor mixing, on the basis of flavor democracy for charged leptons and mass degeneracy for neutrinos [12]. We concentrate on three different symmetry-breaking scenarios in the scheme of this model, which can lead to the “nearly bi-maximal” mixing [12, 13], “small versus large” mixing [14] and “exactly bi-maximal” mixing [15] for solar and atmospheric neutrino oscillations, respectively. Implications of these ansätze, in particular on the upcoming long-baseline neutrino experiments, are also discussed.

4.2 Three-neutrino mixing

The hierarchy of Δm_{21}^2 and Δm_{32}^2 (or Δm_{31}^2) set in Eq. (1) has little implication on the relative magnitude of m_1 , m_2 and m_3 . For this reason, one has to assume specific neutrino mass spectra in constructing phenomenological models of lepton flavor mixing. Two distinct possibilities are of particular interest and have attracted a lot of attention:

- (a) Three neutrino masses perform a clear hierarchy: $m_1 \ll m_2 \ll m_3$;
- (b) Three neutrino masses are almost degenerate: $m_1 \approx m_2 \approx m_3$.

In comparison with the well-known mass spectrum of the charged leptons ($m_e \ll m_\mu \ll m_\tau$), we expect that the charged lepton and neutrino mass matrices (M_l and M_ν) in scenario (a) may both take the hierarchical form, similar to the quark mass matrices. The neutrino mass matrix in scenario (b), however, must take a form quite different from the charged lepton mass matrix. Therefore large mismatch between M_l and M_ν in scenario (b), which leads in most cases to large flavor mixing, seems very natural. In addition, scenario (b) is welcome by the interpretation of hot dark matter, if the sum of three neutrino masses amounts to a few eV [16].

It proves convenient to introduce a specific parameterization for the 3×3 lepton flavor mixing matrix V , which can be obtained from diagonalization of M_l and M_ν in a chosen basis of flavor space. In view of our wealthy knowledge on quark mass matrices and quark flavor mixing, we expect that an appropriate description of lepton flavor mixing, in terms of three Euler angles (θ_l , θ_ν , θ) and one CP -violating phase ϕ , takes the form [13, 17]

$$\begin{aligned}
 V &= \begin{pmatrix} c_l & s_l & 0 \\ -s_l & c_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_\nu & -s_\nu & 0 \\ s_\nu & c_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} s_l s_\nu c + c_l c_\nu e^{-i\phi} & s_l c_\nu c - c_l s_\nu e^{-i\phi} & s_l s \\ c_l s_\nu c - s_l c_\nu e^{-i\phi} & c_l c_\nu c + s_l s_\nu e^{-i\phi} & c_l s \\ -s_\nu s & -c_\nu s & c \end{pmatrix} , \tag{4.2}
 \end{aligned}$$

where $s_l \equiv \sin \theta_l$, $s_\nu \equiv \sin \theta_\nu$, $c \equiv \cos \theta$, etc. Throughout this work we assume CP invariance in the lepton sector, i.e., we take $\phi = 0$. The mixing angles of this parameterization may have very instructive meanings, as one can see later on. In correspondence with the one dominant mass scale approximation made in Eq. (1) for solar and atmospheric neutrino oscillations, $|\theta_l| \ll 1$ is naturally expected. Then the mixing factors $\sin^2 2\theta_{\text{atm}}$ and $\sin^2 2\theta_{\text{sun}}$ turn out to be

$$\begin{aligned}
 \sin^2 2\theta_{\text{atm}} &\approx \sin^2 2\theta , \\
 \sin^2 2\theta_{\text{sun}} &\approx \sin^2 2\theta_\nu , \tag{4.3}
 \end{aligned}$$

to a good degree of accuracy.

It is not difficult to understand the smallness of the mixing angles θ_l , which comes primarily from mixing between the first and second families of the charged lepton mass matrix M_l . In some reasonable Fritzsch-like ansätze of lepton mass matrices (with or without the see-saw mechanism [18, 19]), θ_l is essentially given by

$$\theta_l = \arctan \left(\sqrt{\frac{m_e}{m_\mu}} \right) \approx 4^\circ. \quad (4.4)$$

Therefore the decoupling condition for the solar and atmospheric neutrino oscillations can be satisfied. It should be noted, however, that non-vanishing θ_l may have significant effect on transitions of ν_e and $\bar{\nu}_e$ neutrinos in the long-baseline neutrino experiments.

For example, one may carry out a long-baseline (LB) experiment in which neutrino oscillations are governed only by the mass scale $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2 \approx \Delta m_{31}^2 \gg \Delta m_{21}^2$. In this case the transition probability of ν_μ to ν_e reads

$$P(\nu_\mu \rightarrow \nu_e)_{\text{LB}} = \sin^2 2\theta_l \sin^4 \theta \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{|\mathbf{P}|} \right), \quad (4.5)$$

where \mathbf{P} denotes the momentum of the neutrino beam (in unit of GeV), and L is the distance between the neutrino production and detection points (in unit of km). We observe that $\sin^2 2\theta_l$, like $\sin^2 2\theta$ and $\sin^2 2\theta_\nu$ in Eq. (3), does get an apparent meaning from neutrino oscillations. The transition probability of ν_e to ν_τ in the similar long-baseline experiment is given by

$$P(\nu_e \rightarrow \nu_\tau)_{\text{LB}} = (1 + \tan^2 \theta_l) \cot^2 \theta P(\nu_\mu \rightarrow \nu_e)_{\text{LB}}. \quad (4.6)$$

As a result, measurable signals in Eqs. (5) and (6) (or one of them) in the future experiments will definitely rule out the “exactly bi-maximal” mixing ansatz of three neutrinos [15], in which $\theta_l = 0$ holds.

The question is now of whether the mixing angle θ_ν depends on the neutrino mass ratios. In scenario (a) with $m_1 \ll m_2 \ll m_3$, this m_i -dependence for θ_ν (and also for θ) is naturally expected, leading to the correlation between two oscillation parameters (e.g., between Δm_{21}^2 and $\sin^2 2\theta_\nu$ in the solar neutrino oscillations). For scenario (b) such kind of m_i -dependence might not be welcome, since large cancellation between two almost degenerate neutrino masses is unavoidable in calculating θ_ν and θ . For this reason, we like to follow an approach in which the lepton flavor mixing angles do not depend on the neutrino masses. This is another strategy for our subsequent introduction about a phenomenological model of lepton masses and flavor mixing.

4.3 Nearly bi-maximal mixing

The dominance of m_τ in the hierarchical mass spectrum of charged leptons implies a plausible limit in which the mass matrix takes the form

$$M_l = c_l \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.7)$$

with $c_l = m_\tau$. This mass matrix is equivalent to the following matrix with $S(3)_L \times S(3)_R$ symmetry or flavor democracy:

$$M_{0l} = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad (4.8)$$

through the orthogonal transformation $U_0^\dagger M_l U_0 = M_{0l}$, where

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (4.9)$$

For either M_l or M_{0l} , further symmetry breaking terms can be introduced to generate masses for muon and electron. Recently the possible significance of the approximate democratic mass matrices has been remarked towards understanding fermion masses and flavor mixing [20].

The similar picture is however invalid for the neutrino sector, if three neutrino masses are almost degenerate. Provided the neutrino masses persist in an exact degeneracy symmetry, the corresponding mass matrix should take the diagonal form [21, 22]:

$$M_\nu = c_\nu \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_2 & 0 \\ 0 & 0 & \eta_3 \end{pmatrix}, \quad (4.10)$$

where $c_\nu \equiv m_0 = |m_i|$ (for $i = 1, 2, 3$) measures the mass scale of three neutrinos, and $\eta_i = \pm 1$ (denoting the CP -parity if neutrino masses are of the Majorana type). In the case of $\eta_1 = \eta_2 = \eta_3$, M_ν becomes $M_{0\nu}$ which has exact $S(3)$ symmetry and $m_i = m_0$:

$$M_{0\nu} = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4.11)$$

By breaking the mass degeneracy of M_ν or $M_{0\nu}$ slightly, one may get the realistic neutrino masses m_1 , m_2 and m_3 (at least two of them are different from m_0 and different from each other).

A purely phenomenological assumption is that the fundamental theory of lepton interactions might simultaneously accommodate the charged lepton mass matrix M_{0l} and the neutrino mass matrix $M_{0\nu}$ (or more generally, M_ν) in a specific basis of flavor space. Of course there is no flavor mixing in this symmetry limit. The realistic lepton mass spectra and the flavor mixing matrix depend on the explicit introduction of perturbative corrections to M_{0l} and $M_{0\nu}$. To avoid the dependence of flavor mixing angles on m_i , however, only the diagonal perturbations or the special off-diagonal perturbations to $M_{0\nu}$ are allowed.

We first describe an instructive ansatz with the diagonal perturbations to $M_{0\nu}$ (or M_ν). The first step for symmetry breaking is to introduce small corrections to the (3,3)

elements of M_{0l} and $M_{0\nu}$. The resultant mass matrices read [23]

$$\begin{aligned} M_{1l} &= \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 + \varepsilon_l \end{pmatrix}, \\ M_{1\nu} &= c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + \varepsilon_\nu \end{pmatrix}, \end{aligned} \quad (4.12)$$

where $|\varepsilon_l| \ll 1$ and $|\varepsilon_\nu| \ll 1$. Now the charged lepton mass matrix ceases to be of rank one, and the muon becomes massive ($m_\mu = 2|\varepsilon_l|m_\tau/9$ to the leading order of ε_l). The neutrino mass m_3 is no more degenerate with m_1 and m_2 (i.e., $|m_3 - m_0| = m_0|\varepsilon_\nu|$). It is easy to see, after the diagonalization of M_{1l} and $M_{1\nu}$, that the second and third lepton families have a definite flavor mixing angle θ . We obtain $\tan \theta = -\sqrt{2}$ if the small correction of $O(m_\mu/m_\tau)$ is neglected. Then neutrino oscillations at the atmospheric scale arise in $\nu_\mu \leftrightarrow \nu_\tau$ transitions with the mass-squared difference $\Delta m_{32}^2 = \Delta m_{31}^2 \approx 2m_0|\varepsilon_\nu|$ and the mixing factor $\sin^2 2\theta \approx 8/9$. Such a result is in good agreement with current data.

The next step is to introduce small perturbations to the (2,2) and (or) (1,1) elements of M_{1l} and $M_{1\nu}$, in order to generate the electron mass and to lift the degeneracy between m_1 and m_2 . It has been argued in Refs. [12, 14], in analogy to the quark case, that at this step a simple and instructive perturbation to M_{1l} should be of the form that its (1,1) and (2,2) elements simultaneously receive small corrections of the same magnitude and of the opposite sign. The analogous correction can be introduced to $M_{1\nu}$. Then the lepton mass matrices become [13]

$$\begin{aligned} M_{2l} &= \frac{c_l}{3} \begin{pmatrix} 1 - \delta_l & 1 & 1 \\ 1 & 1 + \delta_l & 1 \\ 1 & 1 & 1 + \varepsilon_l \end{pmatrix}, \\ M_{2\nu} &= c_\nu \begin{pmatrix} 1 - \delta_\nu & 0 & 0 \\ 0 & 1 + \delta_\nu & 0 \\ 0 & 0 & 1 + \varepsilon_\nu \end{pmatrix}, \end{aligned} \quad (4.13)$$

where $|\delta_l| \ll 1$ and $|\delta_\nu| \ll 1$. One finds $m_e = |\delta_l|^2 m_\tau^2 / (27m_\mu)$ to the leading order as well as $m_1 = m_0(1 - \delta_\nu)$ and $m_2 = m_0(1 + \delta_\nu)$. The diagonalization of M_{2l} and $M_{2\nu}$ leads to a full 3×3 flavor mixing matrix, given as U_0 in Eq. (9) if small corrections of $O(\sqrt{m_e/m_\mu})$ and $O(m_\mu/m_\tau)$ are neglected. Then the solar neutrino deficit can be interpreted by $\nu_e \leftrightarrow \nu_\mu$ oscillations with the mass-squared difference $\Delta m_{21}^2 \approx 4m_0|\delta_\nu|$ and the maximal oscillation amplitude [12].

If the corrections from non-vanishing muon and electron masses are taken into account, the lepton flavor mixing matrix will in general read as $V = O_l U_0$, where O_l is an orthogonal matrix. Three rotation angles of O_l are functions of the mass ratios m_e/m_μ and m_μ/m_τ . Due to the strong hierarchy of the charged lepton mass spectrum [10], i.e.,

$$\begin{aligned} \alpha &\equiv \sqrt{\frac{m_e}{m_\mu}} \approx 0.0695, \\ \beta &\equiv \frac{m_\mu}{m_\tau} \approx 0.0594, \end{aligned} \quad (4.14)$$

O_l is expected not to deviate much from the unity matrix. In our specific symmetry-breaking case discussed above, we obtain

$$O_l = \begin{pmatrix} 1 - \frac{1}{2}\alpha^2 & \alpha & \sqrt{2} \alpha\beta \\ -\alpha & 1 - \frac{1}{2}\alpha^2 - \frac{1}{4}\beta^2 & -\frac{1}{\sqrt{2}}\beta \\ -\frac{3}{\sqrt{2}}\alpha\beta & \frac{1}{\sqrt{2}}\beta & 1 - \frac{1}{4}\beta^2 \end{pmatrix} \quad (4.15)$$

to the next-to-leading order. Note that there is another solution for O_l and it can directly be obtained from Eq. (15) with the replacements $\alpha \rightarrow -\alpha$ and $\beta \rightarrow -\beta$. The lepton flavor mixing matrix turns out to be

$$V_{(\pm)} = U_0 \pm (\alpha A - \beta B) - (\alpha^2 C - \alpha\beta D + \beta^2 E) , \quad (4.16)$$

in which A, \dots, E are constant matrices and their explicit forms can be found in Ref. [13] (or read off directly from the product of O_l and U_0). The effects of $O(\alpha^2)$, $O(\alpha\beta)$ and $O(\beta^2)$ on neutrino oscillations will be discussed subsequently.

The flavor mixing matrix obtained above can be parameterized as that in Eq. (2). Under CP invariance we are left with only three Euler angles. We then obtain $\tan \theta_l = 0$, $\tan \theta_\nu = 1$ and $\tan \theta = -\sqrt{2}$ in the limit where terms of $O(\alpha)$ and $O(\beta)$ are neglected. Taking small corrections of $O(\alpha)$ and $O(\beta)$ into account, we arrive at $\tan \theta_l = \pm\alpha$, $\tan \theta_\nu = 1$ and $\tan \theta = -\sqrt{2} (1 \pm 3\beta/2)$, where the “ \pm ” signs correspond to $V_{(\pm)}$ in Eq. (16). The full next-to-leading-order results for three mixing angles are found to be

$$\begin{aligned} \tan \theta_l &= \pm \alpha \left(1 \mp \frac{3}{2}\beta \right) , \\ \tan \theta_\nu &= 1 - 3\sqrt{3} \alpha\beta , \\ \tan \theta &= -\sqrt{2} \left(1 \pm \frac{3}{2}\beta \right) . \end{aligned} \quad (4.17)$$

One can see that the rotation angle θ_ν only receives a tiny correction from the charged lepton masses.

Following Eq. (1) we take $\Delta m_{\text{sun}}^2 = \Delta m_{21}^2$ and $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2 \approx \Delta m_{31}^2$ to accommodate current data on solar and atmospheric neutrino oscillations. Calculating the survival probability $P(\nu_e \rightarrow \nu_e)$ and the transition probability $P(\nu_\mu \rightarrow \nu_\tau)$ to the next-to-leading order, we arrive at [13]

$$\begin{aligned} \sin^2 2\theta_{\text{sun}} &= 1 - \frac{8}{3}\alpha^2 , \\ \sin^2 2\theta_{\text{atm}} &= \frac{8}{9} (1 \mp \beta) . \end{aligned} \quad (4.18)$$

This is just the “nearly bi-maximal” mixing pattern, first proposed in Ref. [12]. The numerical results for mixing angles in Eqs. (17) and (18) are listed in Table 1. One can see that the flavor mixing patterns “ $V_{(+)}$ ” and “ $V_{(-)}$ ” are both consistent with the present data on atmospheric neutrino oscillations. For solar neutrino oscillations, our result favor the “Just-So” solution.

The near degeneracy of three neutrino masses assumed above leads to

$$\left| \frac{m_2 - m_1}{m_3 - m_2} \right| \approx \frac{\Delta m_{\text{sun}}^2}{\Delta m_{\text{atm}}^2} \sim 10^{-7} \quad (\text{“Just-So” solution}) . \quad (4.19)$$

Table 4.1: Numerical results for mixing angles θ_l , θ_ν , θ and $\sin^2 2\theta_{\text{sun}}$, $\sin^2 2\theta_{\text{atm}}$ in the “nearly bi-maximal” mixing scenario [12, 13].

Case	θ_l	θ_ν	θ	$\sin^2 2\theta_{\text{sun}}$	$\sin^2 2\theta_{\text{atm}}$
“ $V_{(+)}$ ”	+3.6°	44.4°	−57.0°	0.99	0.84
“ $V_{(-)}$ ”	−4.3°	44.4°	−52.2°	0.99	0.94

This kind of neutrino mass spectrum can account for the hot dark matter of the universe, if $m_i \approx 2$ eV (for $i = 1, 2, 3$). The relatively large gap between Δm_{21}^2 and Δm_{32}^2 (or Δm_{31}^2) has some implications on the forthcoming long-baseline experiments.

Now we consider the effect of non-vanishing θ_l on the survival probability of electron neutrinos in a long-baseline (LB) experiment, in which the oscillation associated with the mass-squared difference Δm_{21}^2 can be safely neglected due to $\Delta m_{12}^2 \ll \Delta m_{32}^2 \approx \Delta m_{31}^2$. It is easy to find

$$P(\nu_e \rightarrow \nu_e)_{\text{LB}} = 1 - \frac{8}{3}\alpha^2 (1 \mp 2\beta) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{|\mathbf{P}|} \right), \quad (4.20)$$

The oscillation amplitude amounts to 1.1% (the “ $V_{(+)}$ ” case) or 1.4% (the “ $V_{(-)}$ ” case) and might be detectable. One can see that the small mixing obtained here lies well within the allowed region of $\sin^2 2\theta_{\text{CH}}$ from the CHOOZ experiment [7]. The transition probability of ν_μ to ν_e in such a long-baseline neutrino experiment reads

$$P(\nu_\mu \rightarrow \nu_e)_{\text{LB}} = \frac{16}{9}\alpha^2 (1 \mp \beta) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{|\mathbf{P}|} \right). \quad (4.21)$$

Here the mixing factor is about 0.8% (the “ $V_{(+)}$ ” case) or 0.9% (the “ $V_{(-)}$ ” case). The proposed K2K experiment is expected to have a sensitivity of $\sin^2 2\theta > 10\%$ for $\nu_e \leftrightarrow \nu_\mu$ oscillations, while the MINOS experiment could probe values of the mixing as low as $\sin^2 2\theta = 1\%$ [24]. Thus a test of or a constraint on the prediction obtained in Eqs. (20) and (21) would be available in such experiments. The K2K experiment can definitely measure the survival probability of ν_μ neutrinos, which reads as

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{LB}} = 1 - \frac{8}{9}(1 \mp \beta) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{|\mathbf{P}|} \right). \quad (4.22)$$

The mixing factor, corresponding to two different perturbative corrections of the magnitude $\beta \sim 6\%$, takes the value 0.84 or 0.94 (see Table 1). It is also worth mentioning that the transition probability of $\nu_e \rightarrow \nu_\tau$, which satisfies the sum rule

$$P(\nu_e \rightarrow \nu_e)_{\text{LB}} + P(\nu_e \rightarrow \nu_\mu)_{\text{LB}} + P(\nu_e \rightarrow \nu_\tau)_{\text{LB}} = 1, \quad (4.23)$$

is smaller (with the mixing factor $8\alpha^2/9 \approx 0.4\%$) and more difficult to detect.

In the discussions made above the type of neutrinos was not specified. If they are of the Majorana type, then their masses have to fulfill the bound $\langle m_\nu \rangle < 0.45$ eV (at the 90% confidence level) from the neutrinoless $\beta\beta$ -decay [25], where $\langle m_\nu \rangle$ is an effective mass factor. The magnitude of $\langle m_\nu \rangle$ depends both on the neutrino masses m_i and the flavor mixing elements V_{ei} : $\langle m_\nu \rangle = \sum (\eta_i m_i V_{ei}^2)$ for $i = 1, 2$ and 3 , where η_i is the CP parity of the i -th Majorana neutrino field as illustrated in Eq. (10). Taking $\eta_1 = -\eta_2$, one finds that the first and second terms in $\langle m_\nu \rangle$ cancel essentially with each other due to the high degeneracy between m_1 and m_2 as well as the approximate equality between V_{e1}^2 and V_{e2}^2 . The magnitude of $\langle m_\nu \rangle$ turns out to be

$$\langle m_\nu \rangle \approx \frac{2}{\sqrt{3}} \alpha m_i. \quad (4.24)$$

We then arrive at $m_i \approx 12.4 \langle m_\nu \rangle < 5.6$ eV, a bound well within our original expectation $m_i \sim 2$ eV, which can account for the hot dark matter of the universe.

An interesting theoretical understanding of our “nearly bi-maximal” mixing ansatz has recently been made by Mohapatra and Nussinov on the basis of a left-right symmetric extension of the standard model with $S(3)$ and $Z(4) \times Z(3) \times Z(2)$ symmetries [26].

4.4 The “small versus large” mixing scenario

Now we turn to a different symmetry-breaking scenario for the charged lepton mass matrix M_{0l} and the neutrino mass matrix $M_{0\nu}$. Keeping the diagonal symmetry-breaking chain $M_{0l} \rightarrow M_{1l} \rightarrow M_{2l}$ unchanged, we introduce the off-diagonal perturbations to $M_{0\nu}$. To ensure the “maximal calculability” for the neutrino mass matrix, we require a special form of the perturbative matrix: it only has two unknown parameters (to break the mass degeneracy of $M_{0\nu}$) and can be diagonalized by a *constant* orthogonal transformation (independent of the neutrino masses). Then we are left with only three perturbative patterns satisfying these strong requirements, as listed in Table 2. They can be diagonalized by three Euler rotation matrices R_{ij} with the rotation angles $\theta_{ij} = 45^\circ$ in the (1,2), (2,3) and (3,1) planes respectively (see also Table 2). In Ref. [14] pattern (I) was first proposed and discussed. For each pattern the resultant flavor mixing matrix reads as $V = O_l U_0 R_{ij}$, where U_0 and O_l have been given in Eqs. (9) and (15). To the leading order ($\alpha = \beta = 0$), V remains a constant matrix as shown in Table 2. We calculate the transition probability for $\nu_\mu \rightarrow \nu_\tau$ and find the mixing factors to be $8/9$, $2/9$ and $2/9$, corresponding to patterns (I), (II) and (III). Therefore only pattern (I) can survive when confronting the atmospheric neutrino data.

Let us discuss the consequences of pattern (I) in some detail. Note that $\eta_1 = \eta_2 = \eta_3 = 1$ has definitely been taken [14]. Three neutrino masses are given by $m_1 = m_0(1 - \varepsilon_\nu)$, $m_2 = m_0(1 + \varepsilon_\nu)$ and $m_3 = m_0(1 + \delta_\nu)$, respectively. To the next-to-leading order the flavor mixing matrix V has two slightly different forms, because of two possible solutions of O_l . Note that in Ref. [14] only one solution was numerically given. Here we obtain the analytical results of three mixing angles for both possibilities:

$$\tan \theta_l = \pm \alpha \left(1 \mp \frac{3}{2} \beta \right),$$

Table 4.2: Three perturbative patterns for $M_{0\nu}$ and their consequences on V in the leading order approximation ($\alpha = \beta = 0$).

Pattern	Perturbation	Rotation R_{ij}	Flavor mixing V
(I)	$\begin{pmatrix} 0 & \varepsilon_\nu & 0 \\ \varepsilon_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$
(II)	$\begin{pmatrix} \delta_\nu & 0 & 0 \\ 0 & 0 & \varepsilon_\nu \\ 0 & \varepsilon_\nu & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{6}} & \frac{3}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix}$
(III)	$\begin{pmatrix} 0 & 0 & \varepsilon_\nu \\ 0 & \delta_\nu & 0 \\ \varepsilon_\nu & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & -\frac{1}{2} \\ -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{3}{2\sqrt{3}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \end{pmatrix}$

$$\begin{aligned} \tan \theta_\nu &= -\frac{3\sqrt{3}}{2}\alpha\beta \left(1 \mp \frac{1}{2}\beta\right) , \\ \tan \theta &= -\sqrt{2} \left(1 \pm \frac{3}{2}\beta\right) . \end{aligned} \quad (4.25)$$

Comparing these results with those obtained in Eq. (17), one can see that only the value of $\tan \theta_\nu$ changes. Accordingly the mixing factors of solar and atmospheric neutrino oscillations read:

$$\begin{aligned} \sin^2 2\theta_{\text{sun}} &= \frac{4}{3}\alpha^2 (1 \pm 4\beta) , \\ \sin^2 2\theta_{\text{atm}} &= \frac{8}{9} (1 \mp \beta) , \end{aligned} \quad (4.26)$$

to the next-to-leading order. This is just the so-called “small versus large” mixing scenario. The numerical results for mixing angles in Eqs. (25) and (26) are listed in Table 3. We observe that this ansatz favors the small-angle MSW solution to the solar neutrino problem. Its consequence on the atmospheric neutrino oscillations is the same as that obtained in the last section.

It is remarkable that this ansatz also has the same implications on the long-baseline neutrino experiments at the atmospheric scale. This can be seen clearly from Eqs. (5) and (6), in which the $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$ transition probabilities are absolutely independent of the mixing angle θ_ν . Therefore the results obtained already in Eqs. (20) - (22) remain valid in the “small versus large” mixing scenario (for a numerical study of these long-baseline transitions, see Ref. [27]).

Table 4.3: Numerical results for mixing angles θ_l , θ_ν , θ and $\sin^2 2\theta_{\text{sun}}$, $\sin^2 2\theta_{\text{atm}}$ to the next-to-leading order in the “small versus large” mixing scenario.

Case	θ_l	θ_ν	θ	$\sin^2 2\theta_{\text{sun}}$	$\sin^2 2\theta_{\text{atm}}$
“ $V_{(+)}$ ”	+3.6°	−0.60°	−57.0°	8.0×10^{-3}	0.84
“ $V_{(-)}$ ”	−4.3°	−0.63°	−52.2°	4.9×10^{-3}	0.94

If neutrinos are of the Majorana type, then the smallness of both $|\theta_l|$ and $|\theta_\nu|$ in this ansatz implies that the effective mass factor of the neutrinoless $\beta\beta$ -decay (i.e., $\langle m_\nu \rangle$) is dominated by m_1 . A strong constraint turns out to be $m_1 \approx \langle m_\nu \rangle < 0.45$ eV. In addition, the near degeneracy of three neutrino masses assumed above leads to

$$\left| \frac{m_2 - m_1}{m_3 - m_2} \right| \approx \frac{\Delta m_{\text{sun}}^2}{\Delta m_{\text{atm}}^2} \sim 10^{-3}. \quad (4.27)$$

Therefore the sum of three neutrino masses has an upper bound of 1.4 eV, which seems difficult (if not impossible) to account for the hot dark matter of the universe.

4.5 Bi-maximal mixing

Finally let us give some comments on the “exactly bi-maximal” mixing scenario of three neutrinos, which is recently proposed by Barger et al [15]. The relevant flavor mixing matrix, similar to U_0 in Eq. (9), reads as follows:

$$V' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4.28)$$

This flavor mixing pattern is independent of any lepton mass and leads exactly to $\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{\text{sun}} = 1$ for neutrino oscillations. Therefore it favors the “Just-So” solution of the solar neutrino problem. We find that V' can be derived from the following charged lepton and neutrino mass matrices [13]:

$$\begin{aligned} M'_l &= \frac{c'_l}{2} \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \delta'_l & 0 & 0 \\ 0 & 0 & \varepsilon'_l \\ 0 & \varepsilon'_l & 0 \end{pmatrix} \right], \\ M'_\nu &= c'_\nu \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \varepsilon'_\nu & 0 \\ \varepsilon'_\nu & 0 & 0 \\ 0 & 0 & \delta'_\nu \end{pmatrix} \right], \end{aligned} \quad (4.29)$$

where $|\delta'_{l,\nu}| \ll 1$ and $|\varepsilon'_{l,\nu}| \ll 1$. In comparison with the democratic mass matrix M_{0l} given in Eq. (8), which is invariant under the $S(3)_L \times S(3)_R$ transformation, the matrix

M'_l in the limit $\delta'_l = \varepsilon'_l = 0$ only has the $S(2)_L \times S(2)_R$ symmetry. However M'_ν in the limit $\delta'_\nu = \varepsilon'_\nu = 0$ takes the same form as $M_{0\nu}$ in Eq. (11), which displays the $S(3)$ symmetry. The off-diagonal perturbation of M'_l allows the masses of three charged leptons to be hierarchical:

$$\{m_e, m_\mu, m_\tau\} = \frac{c'_l}{2} \{|\delta'_l|, |\varepsilon'_l|, 2 + \varepsilon'_l\}. \quad (4.30)$$

We get $c'_l = m_\mu + m_\tau \approx 1.88$ GeV, $|\varepsilon'_l| = 2m_\mu/(m_\mu + m_\tau) \approx 0.11$ and $|\delta'_l| = 2m_e/(m_\mu + m_\tau) \approx 5.4 \times 10^{-4}$. The off-diagonal perturbation of M'_ν makes three neutrino masses non-degenerate:

$$\{m_1, m_2, m_3\} = c'_\nu \{1 + \varepsilon'_\nu, 1 - \varepsilon'_\nu, 1 + \delta'_\nu\}. \quad (4.31)$$

Taking $\Delta m_{\text{sun}}^2 = \Delta m_{21}^2$ and $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2 \approx \Delta m_{31}^2$ for solar and atmospheric neutrino oscillations, respectively, we then arrive at the same result as that obtained in Eq. (19). The diagonalization of M'_l and M'_ν leads straightforwardly to the flavor mixing matrix V' . In Ref. [15] a different neutrino mass matrix has *reversely* been derived from the given V' in a flavor basis that the charged lepton mass matrix is diagonal. The emergence of the “exactly bi-maximal” mixing pattern from M'_l and M'_ν in Eq. (29) is, in our point of view, similar to that of the “nearly bi-maximal” mixing pattern from M_{2l} and $M_{2\nu}$ in Eq. (13).

Note that three mixing angles of V' are given as $\theta_l = 0$, $\theta_\nu = 45^\circ$ and $\theta = -45^\circ$. The vanishing θ_l leads to vanishing probabilities for $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ transitions in the long-baseline experiments at the atmospheric scale. This feature makes the “exactly bi-maximal” mixing ansatz experimentally distinguishable from the “nearly bi-maximal” mixing ansatz.

If three neutrinos are of the Majorana type, then the near degeneracy of their masses implies that $\langle m_\nu \rangle \approx m_i$ for the neutrinoless $\beta\beta$ -decay (note that $\eta_1 = \eta_2 = \eta_3$ has been taken in M'_ν). Therefore $m_i < 0.45$ eV and the sum of three neutrino masses seems insufficient to account for the hot dark matter of the universe.

4.6 Conclusions

In summary, we have discussed a simple phenomenological model of lepton mass generation and flavor mixing on the basis of flavor democracy for charged leptons and mass degeneracy for neutrinos. Three instructive symmetry-breaking scenarios of this model lead to three interesting mixing patterns of three neutrinos, i.e., the “nearly bi-maximal” mixing, “small versus large” mixing and “exactly bi-maximal” mixing.

The “nearly bi-maximal” mixing and “exactly bi-maximal” mixing ansätze favor the “Just-So” solution of the solar neutrino problem [1]. It is possible to distinguish between these two scenarios in the future long-baseline neutrino experiments. On the other hand, the “small versus large” mixing ansatz favors the small-angle MSW solution to the solar neutrino problem. Whether the solar neutrino deficit is attributed to the “Just-So” oscillation or the matter-enhanced oscillation can finally be clarified by the Super-Kamiokande experiment and other neutrino experiments (e.g., the SNO experiment [28]) under way.

As we have emphasized before, our purely phenomenological approach can only be “justified” by its consequences. If it is experimentally favored, one may get useful hints towards deeper (theoretical) understanding of the phenomena of neutrino masses and

flavor mixing. Recently a lot of attention has been paid to the textures of lepton mass matrices and their implications on neutrino physics [29], in which some similar symmetry arguments were made. Further attempts in this direction are no doubt desirable.

4.7 Discussion

Giunti: Which is the difference between your approach and that of Kang *et al.*?

Xing: The approaches of Kang *et al.* (see Ref. [29]) and ours start essentially from a similar point of view. The main difference lies in how to introduce perturbative corrections. In their case the resulting flavor mixing depends on neutrino masses, while in our case the independence of flavor mixing on neutrino masses is required from the beginning. The results of both approaches are consistent with current data.

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5 Field theoretic treatment of source lifetime and wave-packet effects in neutrino oscillations — Subhendra Mohanty

In the wave-packet treatments of neutrino oscillations the initial neutrino wave-packet is assumed to be a gaussian with spread σ_x in the initial position, and σ_t the uncertainty in the initial time of production of the particle. For accelerator neutrinos which are produced

from the decay of pions or muons, it is assumed that $\sigma_t = v_\nu \times \tau_{source}$. We show below that the gaussian wave-packet treatment for the case of long lifetime neutrino sources — like muons in the LSND experiment — does not reproduce the quantum mechanics formula for neutrino oscillations.

The propagation amplitude of a Gaussian wave-packet turns out to be [2],

$$\tilde{K}(\mathbf{X}, T; m_a) = \exp\{-iE_a T + i\mathbf{P}_a \cdot (\mathbf{X})\} \exp\left\{-\frac{(\mathbf{X} - \mathbf{v}_a T)^2}{4(\sigma_x^2 + v_a^2 \sigma_t^2)}\right\}, \quad (5.1)$$

where $\mathbf{X} = \mathbf{x}_f - \mathbf{x}_i$, $T = t_f - t_i$ and $(\sigma_x^2 + v_a^2 \sigma_t^2)^{1/2} \equiv \bar{\sigma}$ is the width of the wave-packet.

The probability of flavor oscillation as a function of space-time is

$$P(\alpha \rightarrow \beta; X, T) = \left| \sum_{a=1}^3 U_{\alpha a} \tilde{K}(m_a; X, T) U_{a\beta}^* \right|^2. \quad (5.2)$$

The probability as a function of distance is given by the time average of $P(\alpha \rightarrow \beta; X, T)$:

$$P(\alpha \rightarrow \beta; X) = \int dT \left| \sum_{a=1}^3 U_{\alpha a} \tilde{K}(m_a; X, T) U_{a\beta}^* \right|^2. \quad (5.3)$$

The interference term is

$$\frac{2}{v} \cos \left[\frac{X}{P} (E\Delta E - \mathbf{P} \cdot \Delta \mathbf{P}) \right] e^{-A} = \frac{2}{v} \cos \left(\frac{\Delta m^2}{2P} X \right) e^{-A}, \quad (5.4)$$

where the exponential damping factor A is given by

$$A = (\Delta E)^2 \frac{\bar{\sigma}^2}{2v^2} + \left(\frac{\Delta P}{P} \right)^2 \frac{X^2}{4\bar{\sigma}^2}. \quad (5.5)$$

The specific form of the phase difference $E\Delta E - \mathbf{P} \cdot \Delta \mathbf{P}$ which appears in a covariant calculation has interesting applications. One can show that for this form of phase difference there are no EPR-type associated particle oscillations [2]. Consider a pion decay $\pi \rightarrow \mu\nu$ in the rest frame of the pion. Using the conservation laws

$$\begin{aligned} E_\nu &= \frac{m_\pi^2 - m_\mu^2 + m_\nu^2}{2m_\pi}, \\ E_\mu &= \frac{m_\pi^2 + m_\mu^2 - m_\nu^2}{2m_\pi}, \\ P &= |\mathbf{P}_\nu| = |\mathbf{P}_\mu| = \frac{[(m_\pi^2 - (m_\nu + m_\mu)^2)(m_\pi^2 - (m_\nu - m_\mu)^2)]^{1/2}}{2m_\pi}, \end{aligned}$$

the neutrino phase differences are given by

$$\begin{aligned} \Delta E_\nu &= \frac{\Delta m_\nu^2}{2m_\pi}, \\ 2\Delta P_\nu &= \frac{\Delta m_\nu^2}{2m_\pi P} (E_\nu - m_\pi), \\ \Delta\phi_\nu &= (E_\nu \Delta E_\nu - \mathbf{P}_\nu \cdot \Delta \mathbf{P}_\nu) \frac{X}{P_\nu} = \frac{\Delta m_\nu^2}{2P} X. \end{aligned}$$

The phase differences between different muon states are given by

$$\begin{aligned}\Delta E_\mu &= -\frac{\Delta m_\nu^2}{2m_\pi}, \\ \Delta P_\mu &= -\frac{\Delta m_\nu^2}{2m_\pi P} E_\mu, \\ \Delta\phi_\mu &= (E_\mu \Delta E_\mu - \mathbf{P}_\mu \cdot \Delta \mathbf{P}_\mu) \frac{X}{P_\mu} = 0.\end{aligned}$$

This shows that there are no EPR-type secondary particle oscillations.

Going back to the neutrino oscillation term (5.4), we consider the case

$$\bar{\sigma} \equiv (\sigma_x^2 + v^2 \sigma_t^2)^{1/2} \simeq v\tau. \quad (5.6)$$

When the source-detector distance X is such that

$$X \ll X_{\text{coh}} = \frac{2\bar{\sigma}P}{\Delta P}, \quad (5.7)$$

we have

$$A \simeq 2(\Delta E)^2 \bar{\sigma}^2 \simeq \left(\frac{\Delta m^2 \tau}{2\sqrt{2}E} \right)^2 \quad (5.8)$$

The resulting wave-packet formula for neutrino oscillation is

$$P(\nu_\alpha \rightarrow \nu_\beta; X) = \frac{1}{2} \sin^2 2\theta \left\{ 1 - \cos \left(\frac{2.53 \Delta m^2 X}{E} \right) \exp \left[- \left(\frac{1.79 \Delta m^2 \tau}{E} \right)^2 \right] \right\}. \quad (5.9)$$

In the LSND experiment (neutrinos from muon decay) the source lifetime is $\tau_\mu = 658.6$ m and the exponential factor is significant.

In other accelerator experiments with neutrinos from pion or kaon decay the source lifetimes is ~ 7 m and the exponential factor is negligible.

From Figure 5.1 it is clear that the standard gaussian wave-packet treatments does not reproduce the quantum mechanical formula when the source of the neutrinos has a large lifetime.

Why is the neutrino wave-packet treatment inconsistent?

A spatial width $\sigma_x \sim 659$ m of the neutrinos implies that the spread of neutrino momentum is $\sigma_P = 1/2\sigma_x \sim 10^{-13}$ MeV. This is experimentally unreasonable as the spread of the neutrino momentum must be at least as large as the momentum spread of the source (pion, muon) wave packet, which is expected to be $\sim 1 - 10$ MeV.

To incorporate the source lifetime we must include the Breit-Wigner propagator of the source in the Feynman diagram [3].

We consider the LSND experiment with the process

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \xrightarrow{\nu^{\text{osc.}}} \bar{\nu}_e + p \rightarrow n + e^+ \quad (5.10)$$

as a model for our investigation of the source-lifetime dependence of neutrino oscillations. The amplitude of the process $(\mu^+)_S + (p)_D \rightarrow (e^+ \nu_e)_S + (n + e^+)_D$ is expressed as the

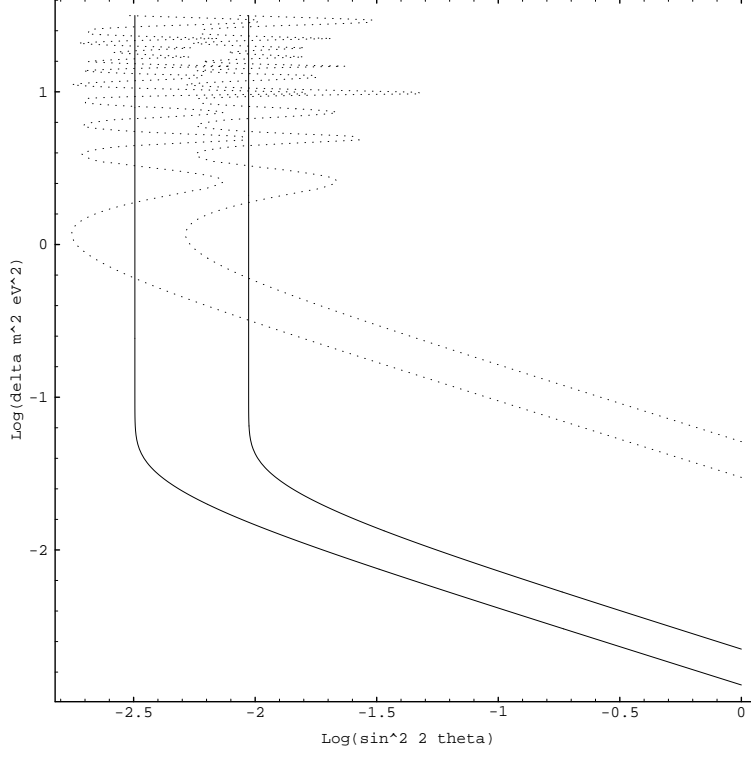


Figure 5.1

linear combination

$$\begin{aligned}
\mathcal{A} &= \sum_j U_{\mu j} U_{ej}^* e^{iq_j L} \mathcal{A}_j \\
&= \frac{G_F^2 \cos \vartheta_C}{2} \frac{2\pi^2}{L} i \sum_j U_{\mu j} U_{ej}^* e^{iq_j L} \frac{1}{i(E_\mu - E_{\nu S} - E_{eS} - E_{\nu_j}) + \frac{1}{2}\Gamma} \\
&\quad \times \bar{\psi}_\mu (\vec{p}_1 + q_j \vec{l}) \gamma_\rho (1 - \gamma_5) (-\vec{k}_j + m_j) \gamma_\lambda (1 - \gamma_5) v_e(p'_{eD}) \\
&\quad \times J_S^p(p'_\nu, p'_{eS}) \bar{u}(p'_n) \gamma^\lambda (1 - g_A \gamma_5) \tilde{\psi}_p(-q_j \vec{l} + \vec{p}_2). \tag{5.11}
\end{aligned}$$

The cross section obtained by taking the modulus square of (5.11) gives the flavor oscillation probability [4]:

$$\begin{aligned}
P_{\nu_\alpha \rightarrow \nu_\beta} &= \sum_j |U_{\beta j}|^2 |U_{\alpha j}|^2 \\
&\quad + \sum_{j>k} U_{\beta j}^* U_{\alpha j} U_{\beta k} U_{\alpha k}^* \exp \left[- \left(\frac{1}{8\sigma_D^2} + \frac{1}{8\sigma_S^2} \right) \left(\frac{\Delta m_{jk}^2}{2E_\nu} \right)^2 \right] \\
&\quad \times \left\{ \frac{\Gamma^2 - \left(\frac{\Delta m_{jk}^2}{2E_\nu} \right)^2}{\Gamma^2 + \left(\frac{\Delta m_{jk}^2}{2E_\nu} \right)^2} \cos \left(\frac{\Delta m_{jk}^2 L}{2E_\nu} \right) + \frac{\Gamma \left(\frac{\Delta m_{jk}^2}{2E_\nu} \right)}{\Gamma^2 + \left(\frac{\Delta m_{jk}^2}{2E_\nu} \right)^2} \sin \left(\frac{\Delta m_{jk}^2 L}{2E_\nu} \right) \right\}. \tag{5.12}
\end{aligned}$$

Final comments:

- In the limit of a small source-lifetime ($\Gamma \gg \Delta E$), which is valid for example for neutrinos from Z decay, the lifetime dependence drops out and we get back the standard oscillation formula.
- The most interesting application of (5.12) is in the regime $\Gamma \leq \Delta E$. In this regime, which includes LSND, the Δm^2 probed by (5.12) is lower than what one obtains from the standard oscillation formula.
- In earlier treatments σ_S and σ_D were taken to be the momentum spread of the neutrino wave-packet, that is a quantity over which there is no experimental control. In this analysis σ_S and σ_D are the spread of the muon and proton momenta which are better known experimentally.

5.1 Discussion

Giunti: I think that the sensitivity to Δm^2 of a neutrino oscillation experiment is determined only by the argument $\Delta m^2 X/E$ of the oscillatory term. If P_{\max} is the maximum value of the oscillation probability measurable in a given experiment, the minimum value of Δm^2 that can be probed is $\Delta m_{\min}^2 \sim \sqrt{P_{\max}} E/X$ (with appropriate averages over energy and distance).

A long lifetime of the source means that the coherence length is very large, but it cannot have any effect on Δm_{\min}^2 . Indeed, a long lifetime of the source implies that the spatial width of the neutrino wave packet is very large and the standard plane-wave treatment is applicable. A correct wave-packet treatment must reproduce this result.

In the case of the LSND experiment the anti-muon does not decay exactly at rest, because it is in a medium and its kinetic energy must be of the order of the thermal energy of the medium. The corresponding velocity is $v_\mu \sim 3 \times 10^{-5}$. The mean free path of the anti-muon in the medium is of the order of the inter-nuclear distance: $\ell_\mu \sim 10^{-8}$ cm. Therefore, the coherent emission of the neutrino wave is interrupted every $\ell_\mu/v_\mu \sim 10^{-14}$ s, which is much smaller than the muon lifetime, $\tau_\mu \simeq 2 \times 10^{-6}$ s.

Furthermore, the positron emitted in the anti-muon decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ is relativistic ($v_e \simeq 1$) and is annihilated as soon as it interacts with an electron in the medium. Approximating its mean free path with the inter-nuclear distance, $\ell_e \sim 10^{-8}$ cm, one can see that the coherence of the μ^+ -decay process is interrupted after $\ell_e/v_e \sim 3 \times 10^{-19}$ s. This is the dominant effect in the determination of the spatial width of the neutrino wave packet, which results to be of the order of 10^{-8} cm, much smaller than the ~ 600 m obtained from the muon lifetime.

Mohanty: I agree that in LSND the muon decay does not happen in vacuum and the correct value for σ_t should be the mean free path of the muon.

My analysis is not specific to LSND but to the general case of neutrinos produced from isolated long lived particles. Then, from the oscillation formula (5.9) one can see that when there is an incoherent superposition (for example when the detector distance is much larger than the coherence length) we have $P = \frac{1}{2} \sin^2 2\theta$. The plot of this probability in the Δm^2 vs $\sin^2 2\theta$ plane gives a vertical line at $\frac{1}{2} \sin^2 2\theta$ for

all values of Δm^2 for which the decoherence condition is satisfied. So the general result is that the incoherent superposition formula rules out a larger portion of the Δm^2 vs $\sin^2 2\theta$ graph than the coherent superposition formula. Therefore, it is of practical interest to examine various ways in which we can obtain an incoherent superposition.

The second point I wanted to make is that the assumption of a gaussian wave-packet with width equal to the lifetime, which is the standard assumption in literature (see Ref. [5] and references therein), does not reproduce the quantum mechanics formula.

Lipkin: There is one problem with the "field theoretic treatment" of the source lifetime. The problem is that the neutrino is not emitted alone from the source; there is also another change in the environment. If we are considering a long-lived beta decay of a nucleus bound in an atom, the nuclear lifetime is irrelevant for neutrino coherence because the nucleus is interacting with the atom, and the atom knows when the charge of the nucleus has changed and an electron or positron has been emitted together with the neutrino.

The point has been repeatedly made by Leo Stodolsky that the proper formalism to treat neutrino oscillations is not field theory but the density matrix, because only in this way the unavoidable interactions with the environment can be taken into account. Leo also points out that the length in time of the wave packet is irrelevant [6].

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6 The Nucleosynthesis Limit on N_ν — Subir Sarkar

Hoyle and Tayler [1] as well as Peebles [2] had emphasized many years ago that new types of neutrinos (beyond the ν_e and ν_μ then known) would boost the relativistic energy density hence the expansion rate² during big bang nucleosynthesis (BBN), thus increasing the yield of ^4He . Shvartsman [3] noted that new superweakly interacting particles would have a similar effect. Subsequently this argument was refined quantitatively by Steigman, Schramm and collaborators [4]. In the pre-LEP era when the laboratory bound

²In the radiation-dominated era, $H = \sqrt{8\pi G_N \rho/3}$, with $\rho = \frac{\pi^2}{30} g_* T^4$ where g_* counts the relativistic degrees of freedom.

on the number of neutrino species was not very restrictive [5], the BBN constraint already indicated that at most one new family was allowed [6], albeit with rather uncertain systematics [7]. Although LEP now finds $N_\nu = 2.991 \pm 0.016$ [8], the cosmological bound is still important since it is sensitive to *any* new light particle, not just $SU(2)_L$ doublet neutrinos, so is a particularly valuable probe of new physics [9].³

The primordial mass fraction $Y_p(^4\text{He})$ increases as $\approx 0.012\Delta N_\nu$ but it also increases logarithmically with the nucleon density (usually parameterized as $\eta \equiv n_N/n_\gamma = 2.728 \times 10^{-8}\Omega_N h^2$). Thus to obtain a bound on N_ν requires an upper limit on Y_p and a lower limit on η . The latter is poorly determined from direct observations of luminous matter so must be derived from the abundances of the other synthesized light elements, D, ^3He and ^7Li , which are power-law functions of η . The complication is that these abundances are substantially altered in a non-trivial manner during the chemical evolution of the galaxy, unlike $Y_p(^4\text{He})$ which just increases by a few percent due to stellar production. (This can be tagged via the correlated abundance of oxygen and nitrogen which are made *only* in stars.)

Even so, some cosmologists have used chemical evolution arguments to limit the primordial abundances of D and ^3He and thus derived increasingly severe bounds on N_ν [10], culminating in a recent one *below* 3 [11]! However a more conservative view [12] is that there is no crisis with BBN if we recognize that such arguments are rather dubious and consider only *direct* measurements [13] of light element abundances, as shown in Figure 1. The ^4He mass fraction is obtained from observations of metal-poor blue compact galaxies by linear extrapolation to zero nitrogen/oxygen abundance [14]; the upper limit is reliable, the lower one less so. In particular, older measurements had suggested a value smaller by about 4% [15]. At present there are two conflicting measurements of the D abundance in quasar absorption systems [16,17]; the higher value [16] is interpreted as an upper limit. Also shown is the abundance in the interstellar medium [18] which provides a reliable lower limit. The ^7Li abundance as measured in the hottest, most metal-poor halo stars [19] as well as in disk stars [20] is shown and interpreted as providing, respectively, reliable lower and upper limits on its primordial value. Given these uncertainties, standard BBN is consistent with observations for $\eta \approx 2 - 9 \times 10^{-10}$. Adopting the reliable limits, $Y_p(^4\text{He}) < 0.25$, $\text{D}/\text{H} > 1.1 \times 10^{-5}$ and $^7\text{Li}/\text{H} < 1.5 \times 10^{-9}$, and taking into account uncertainties in nuclear cross-sections and the neutron lifetime by Monte Carlo, we obtain [12]

$$N_\nu^{\text{max}} = 3.75 + 78 (Y_p^{\text{max}} - 0.240), \quad (6.1)$$

i.e. up to 1.5 additional (equivalent) neutrino species are allowed for η at its lowest allowed value. It is clear that the restrictions on new physics are less severe than had been reported previously [10].

Other workers have applied Bayesian likelihood methods to obtain $N_\nu < 4 - 5$ [21]. In order to enable extraction of the best-fit N_ν/η values as the observational situation improves (and uncertainties in the input nuclear cross-sections decrease), we have developed a simple method for determining the (correlated) uncertainties of the expected abundances [22]. Essentially we compute the full covariance error matrix, having checked the linearity of error propagation, and then perform a simple χ^2 fit to the observational

³The energy density of new light fermions i is equivalent to an effective number $\Delta N_\nu = \sum_i (g_i/2)(T_i/T_\nu)^4$ of additional doublet neutrinos, where T_i/T_ν can be calculated from considerations of their (earlier) decoupling [9].

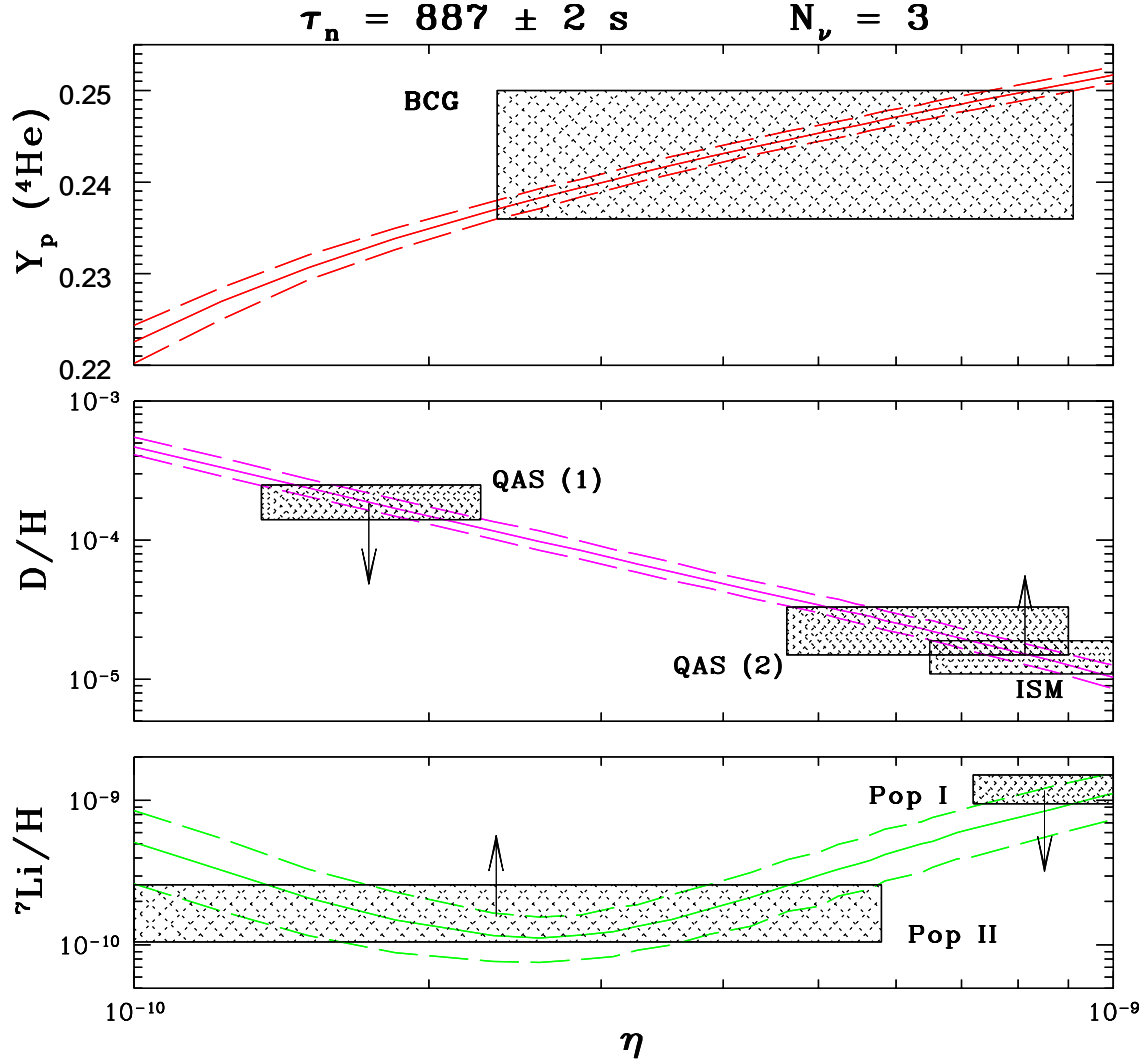


Figure 6.1: Predicted light element abundances for the Standard Model versus the nucleon-to-photon ratio [12]. The 95% c.l. limits determined by Monte Carlo reflect the uncertainties in input nuclear cross-sections and the neutron lifetime. Rectangles indicate observational determinations and associated ‘95% c.l.’ bounds.

data. This requires us to input actual measurements, not limits as above so we consider both the reported (and mutually incompatible) values for the deuterium abundance:

$$D/H = 3.40 \pm 0.25 \times 10^{-5} \quad (\text{low}) \quad (6.2)$$

$$= 1.9 \pm 0.5 \times 10^{-4} \quad (\text{high}), \quad (6.3)$$

as well as for the helium mass fraction:

$$Y_p(^4\text{He}) = 0.234 \pm 0.0054 \quad (\text{low}) \quad (6.4)$$

$$= 0.244 \pm 0.0054 \quad (\text{high}). \quad (6.5)$$

In Figure 2 we show the likelihood contours in the N_ν - η plane for the four possible combinations of the above measurements. (The results are relatively insensitive to the ^7Li

abundance.) We see that both the high D/low ^4He and the low D/high ^4He combinations are consistent with $N_\nu = 3$ and indeed the χ^2 for the fits is excellent [22]. However the low D/high ^4He combination suggests that $N_\nu \simeq 2.3$, while the high D/low ^4He combination suggests that $N_\nu \simeq 3.8$! If either of these possibilities are indeed substantiated by further measurements, then this would indicate a departure from the Standard Model, e.g. the presence of a sterile neutrino which mixes with the left-handed doublets. Interestingly enough, this can both raise and lower the effective value of N_ν [23].

A clear discrimination between these possibilities will be provided when forthcoming precision measurements of small-scale angular anisotropy in the cosmic microwave background [24] provide an independent measure of the nucleon density η to an accuracy of a few per cent [25].

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deuterium and helium

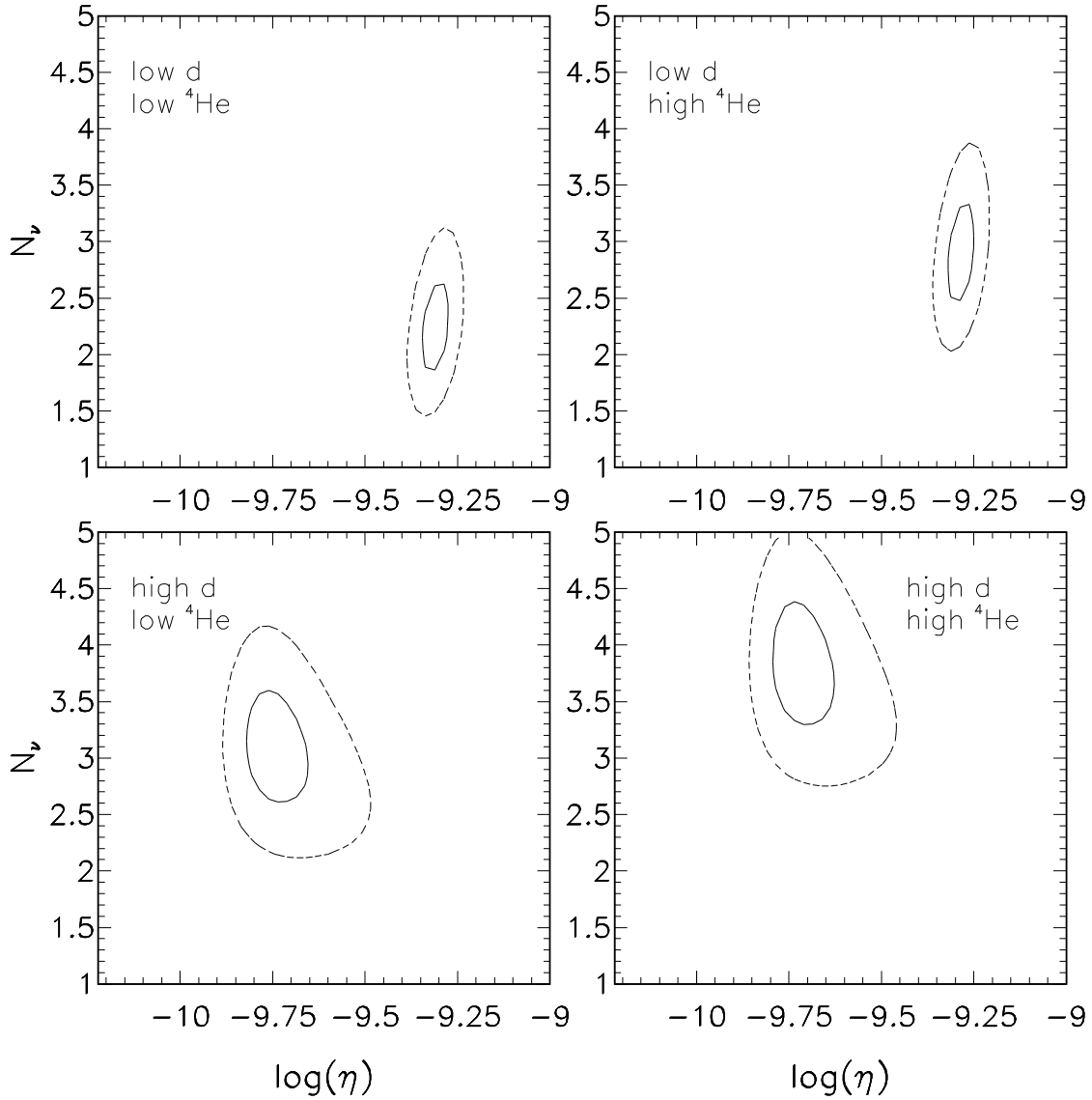


Figure 6.2: The 68% (solid) and 95% (dotted) likelihood contours for the number of neutrino species and the nucleon-to-photon ratio, for all possible combinations of the present (discrepant) deuterium and helium abundance measurements [22].

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7 Are the results of LSND and KARMEN 2 compatible? — Carlo Giunti

The KARMEN collaboration has reported recently [1] a null result of the KARMEN 2 experiment searching for neutrino oscillations in the channel $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. This result is very interesting because the KARMEN neutrino oscillation experiment [2, 3] is sensitive to the same region in the plane of the neutrino mixing parameters $\sin^2 2\theta$ and Δm^2 as the LSND experiment [4] whose results provide an evidence in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ oscillations. Hence, the statistical interpretation of the KARMEN 2 null result (as well as that of the LSND result) is crucial in order to obtain an indication on the compatibility or incompatibility of the different results of the two experiments.

So far the KARMEN 2 experiment measured no events [1], with an expected background of 2.88 ± 0.13 events. This null result has been analyzed with the following statistical methods:

Bayesian Approach. This method is accepted by the Particle Data Group [5, 6] and has been used by the KARMEN collaboration [1]. The resulting upper limit for the mean μ of neutrino oscillation events is 2.3 and the corresponding exclusion curve is shown in Fig. 7.1 (the solid curve passing through the filled squares).

Unified Approach. This frequentist method has been proposed recently by Feldman and Cousins [7]. It is very attractive because it allows to construct classical confidence belts with the correct coverage that “unify the treatment of upper confidence limits for null results and two-sided confidence intervals for non-null results” [7]. On the other hand, the Unified Approach has the undesirable feature that when the number of observed events is smaller than the expected background, the upper limits for the mean μ of true neutrino oscillation events decreases rapidly when the background increases. From a physical point of view this is rather disturbing, because *a stringent upper bound for μ obtained by an experiment which has observed*

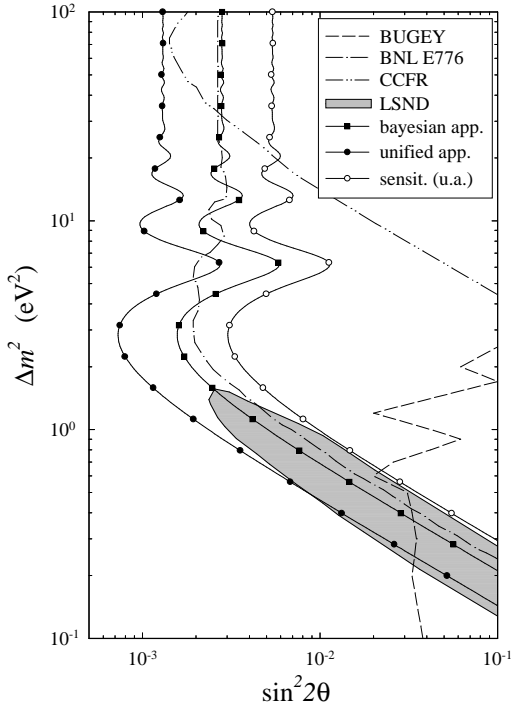


Figure 7.1

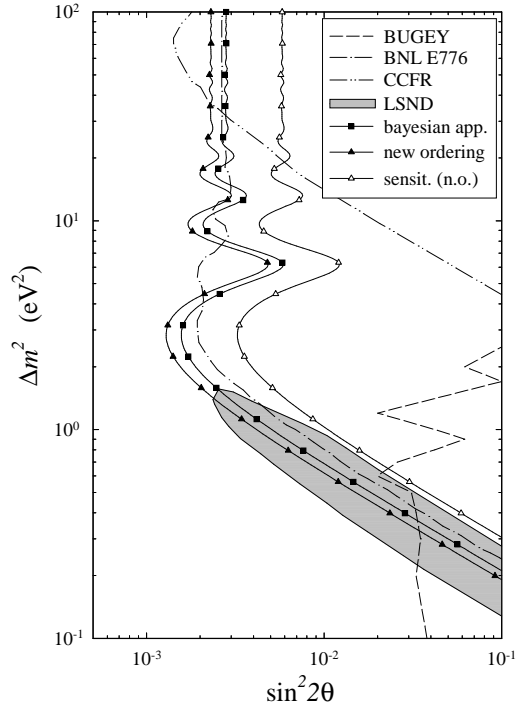


Figure 7.2

a number of events significantly smaller than the expected background is not due to the fact that the experiment is very sensitive to small values of μ , but to the fact that less background events than expected have been observed.

This is the case of the null result of the KARMEN 2 experiment, from which the Unified Approach yields an upper 90% CL confidence limit of 1.1 events for the mean μ of neutrino oscillation events. The corresponding exclusion curve in the plane of the neutrino mixing parameters $\sin^2 2\theta$ and Δm^2 is shown in Fig. 7.1 (the solid curve passing through the filled circles) and one can see that it is significantly more stringent than the exclusion curve obtained with the Bayesian Approach (the solid curve passing through the filled squares). The strictness of the Unified Approach exclusion curve is due to the non-observation of the expected background events and not to the sensitivity of the experiment (see the discussion in Ref. [8] and the sensitivity of the KARMEN experiment presented in Ref. [9]). This is clearly an undesirable result from a physical point of view, because *the statistical interpretation of the data produces an exaggeratedly stringent result that could lead to incorrect physical conclusions.*

The discrepancy between the Bayesian Approach exclusion curve and the Unified Approach exclusion curve is worrying for a physicist, because the Bayesian Approach exclusion curve is compatible with a large part of the LSND-allowed region (the shadowed area in Fig. 7.1), whereas the Unified Approach exclusion curve excludes almost all the LSND allowed region.

In view of the uncertainty of the physical meaning of the KARMEN 2 null result induced by the significant difference between the exclusion curves obtained with the Unified Approach on one hand and with the Bayesian Approach on the other hand, it is inter-

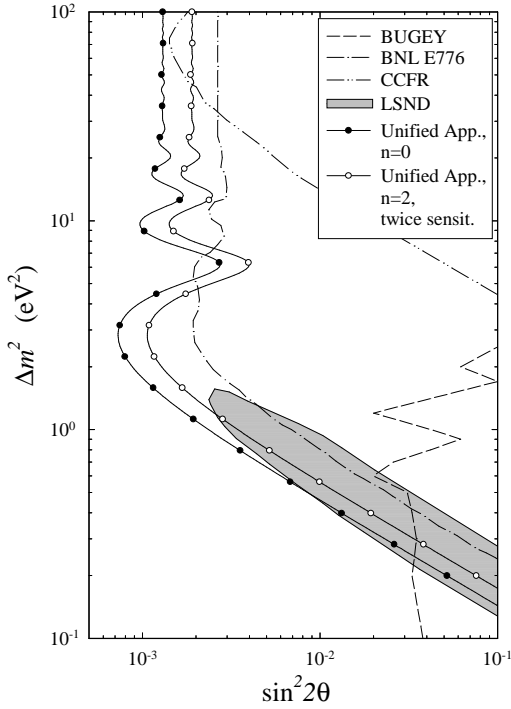


Figure 7.3

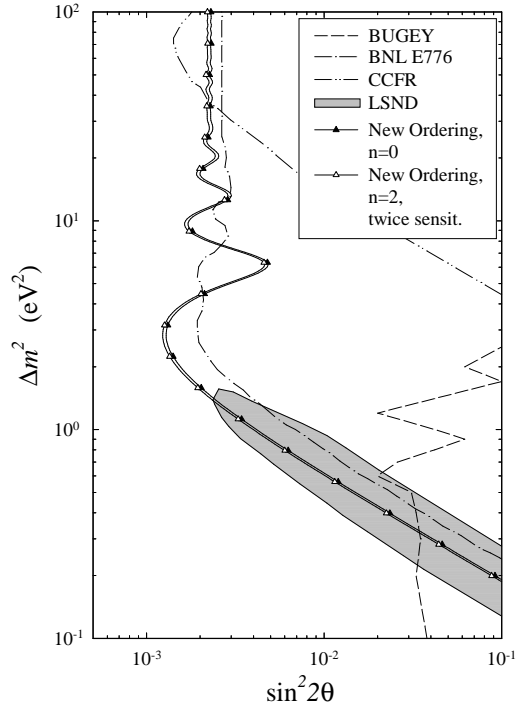


Figure 7.4

esting to explore other possibilities for the statistical interpretation of the KARMEN 2 null result.

In this report I present the outcome of the statistical analysis of the null result of the KARMEN 2 experiment with the New Ordering Approach that has been proposed in Ref. [8]. This approach is based on a new ordering principle for the construction of a classical frequentist confidence belt that has all the desirable properties of the one calculated with the Unified Approach and in addition minimizes the effect on the resulting confidence intervals of the observation of less background events than expected. Hence, it is appropriate for the statistical interpretation of the null result of the KARMEN 2 experiment. The resulting upper limit for the mean μ of true neutrino oscillation events in the KARMEN 2 experiment is 1.9 and the corresponding exclusion curve [8] is shown in Fig. 7.2 (the solid curve passing through the filled triangles).

The exclusion curve obtained with the New Ordering Approach lies close to the bayesian exclusion curve and tends to support the compatibility of the KARMEN 2 and LSND results. This is a desirable achievement. Furthermore, it is important to emphasize that *the New Ordering Approach gives a correct frequentist coverage as the Unified Approach*.

Hence, the New Ordering Approach has solved the apparent conflict between the frequentist and bayesian statistical interpretation of the null result of the KARMEN 2 experiment: *by choosing an appropriate ordering principle in the construction of the confidence belt, the exclusion curve obtained with the frequentist method is in reasonable agreement with the one obtained with the Bayesian Approach*.

The different impact of the non observation of the expected background events on the exclusion curves obtained with the Unified Approach and with the New Ordering Approach is illustrated in Figs. 7.3 and 7.4. In these figures the exclusion curves obtained

from the null result of the KARMEN 2 experiment ($n = 0$, where n is the number of observed events) are compared with the exclusion curves that would be obtained by an imaginary experiment similar to the KARMEN 2 experiment but with a sensitivity twice that of the KARMEN 2 experiment, which observe two events ($n = 2$), in good agreement with the expected mean background $b = 2.88 \pm 0.13$. The results of both experiments (real and imaginary) are compatible with the absence of neutrino oscillations and imply upper bounds for the neutrino oscillation parameters.

One can see from Fig. 7.3 that the imaginary experiment with twice sensitivity would yield a Unified Approach exclusion curve (the solid curve passing through the empty circles) which is significantly *worse* than that obtained from the null result of the KARMEN 2 experiment (the solid curve passing through the filled circles). On the other hand Fig. 7.4 shows that the imaginary experiment with twice sensitivity would yield a New Ordering exclusion curve (the solid curve passing through the empty triangles) which is practically equivalent to that obtained from the null result of the KARMEN 2 experiment (the solid curve passing through the filled triangles). It is clear that the impact of the non observation of the expected background events on the resulting exclusion curve cannot be eliminated completely, but can be minimized with an appropriate construction of the frequentist confidence belt, as that resulting from the New Ordering Principle.

I conclude this report with the following remarks:

1. The statistical interpretation of the null result of the KARMEN 2 neutrino oscillation experiment is rather problematic because no events were observed with a mean expected background of 2.88 ± 0.13 events [1]. The exclusion curves obtained with the Bayesian Approach and with the Unified Approach [7] are significantly different and yield contradicting indications on the compatibility of the KARMEN 2 result with the neutrino oscillation signal measured in the LSND experiment [4] (see Fig. 7.1).
2. The analysis of the KARMEN 2 null result with the New Ordering Approach [8], which is a frequentist method with correct coverage as the Unified Approach, yields an exclusion curve close to the one obtained with the Bayesian Approach (see Fig. 7.2). In this way, the undesirable discrepancy between frequentist and bayesian interpretations of the KARMEN 2 null result is removed.
3. Taking into account the error of the expected mean background $b = 2.88 \pm 0.13$ in the KARMEN 2 experiment, even if it is wrong by an order of magnitude, does not help in solving the problem of the statistical interpretation of the result of this experiment, because the resulting exclusion curves are practically equivalent to the ones obtained assuming no error for the expected mean background [10].
4. An extreme attitude⁴ is to ignore the calculated mean expected background and to assume that the background is unknown. This approach gives ultra-conservative exclusion curves [10], which in the case of the KARMEN 2 experiment tend to

⁴Let me emphasize that this attitude is purely speculative and does not have any justification from the experimental point of view. The background in the KARMEN 2 experiment (except for the component due to the intrinsic $\bar{\nu}_e$ contamination of the beam, that is expected to contribute with 0.56 ± 0.09 events) is measured on-line with high precision in parallel with the neutrino oscillation search [11, 12].

support the exclusion curve obtained with the Bayesian Approach. Obtaining ultra-conservative exclusion curves is generally not desirable, but could be considered to be a safe choice in controversial cases as that of the KARMEN 2 experiment.

5. The sensitivity of an experiment has been defined by Feldman and Cousins as “the average upper limit that would be obtained by an ensemble of experiments with the expected background and no true signal” [7]. The 90% CL sensitivity curves obtained from the null result of the KARMEN 2 experiment with the Unified Approach and with the New Ordering Approach are shown in Figs. 7.1 and 7.2 (the solid curves passing through the empty circles and triangles, respectively). In Ref. [10] it has been shown that *a sensitivity curve cannot be considered as an exclusion curve*. Since the sensitivity curve of a neutrino oscillation experiment can be calculated before doing the experiment, its usefulness lies in the possibility to plan future experiments in order to cover approximately the region of interest in the plane of the neutrino mixing parameters $\sin^2 2\theta$ and Δm^2 . However, a large discrepancy between the exclusion and sensitivity curves is a signal that the exclusion curve results form a rather improbable experimental result and one must be very cautious in formulating physical conclusions on the basis of the exclusion curve. It can be seen from Figs. 7.1 and 7.2 that from this point of view the new ordering approach is safer than the unified approach.
6. The direct comparison of exclusion curves and allowed regions obtained with different statistical methods does not have a precise statistical significance. Hence, such a comparison cannot be used to combine the results of different experiments or to infer with some known confidence level a contradiction between the results of different experiments when the comparison is done on the border of the exclusion curves and of the allowed regions. Hence, the comparison of the KARMEN 2 exclusion curves and the LSND-allowed region, which was obtained with a different statistical analysis (see Ref. [4]), must be done with great caution.
7. It is clear that the null result of KARMEN 2 does not favor the credibility of the positive result of the LSND experiment, but I think that *it is still premature to claim a contradiction between the two experiments*.
8. In the future, if the KARMEN 2 experiment will continue to observe no neutrino oscillations, it will be possible to claim a contradiction between the results of the two experiments only when the exclusion curves obtained from the result of the KARMEN 2 experiment with different statistical methods will produce similar results and will lie well on the left of the region allowed by the results of the LSND experiment.

7.1 Discussion

Sarkar: I think that the Unified Approach exclusion curve is fine, as long as it is interpreted in the correct way.

Giunti: Yes, I agree. However, the majority of physicists are not expert in the subtleties of statistics and tend to believe in the exclusion of the parameter region on the right

of an exclusion curve. This is statistically incorrect and very misleading in some cases, as that of the KARMEN 2 experiment. Therefore, I think that it is desirable to present the experimental results in a way that leaves less space to improper interpretations. I think that this can be done using the New Ordering Principle, which is equivalent to the Unified Approach from the statistical point of view: both of them give an allowed range of the mixing parameters that belongs to a set of allowed ranges that could be obtained with an ensemble of experiments identical to the KARMEN 2 experiment and cover the true values of the mixing parameters with probability 0.90 (for 90% CL exclusion curves).

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